06

In daily life, we generally use the terms work, energy and power. Work is any activity involving mental or physical effort.
Whereas, energy is our capacity to do work. The word power is also used in everyday life with different shades of meaning. In this chapter, we will study these terms from the vision of science.

WORK, ENERGY AND POWER

|TOPIC 1|

Work

The term work as understood in everyday life i.e. any physical or mental labour, has a different meaning in science. If a coolie is carrying a load on his head and waiting for the arrival of the train, he is not performing any work in the scientific sense.

CHAPTER CHECKLIST

- Work
- Conservative Force
- Non-conservative Force
- Kinetic Energy
- Work Energy Theorem
- Potential Energy
- Various Forms of Energy
- Power
- Collision

DEFINITION OF WORK

Work is said to be done by a force, when the body is displaced actually through some distance in the direction of the applied force. Thus, work is done on a body only if the following two conditions are satisfied:

(i) A force acts on the body.

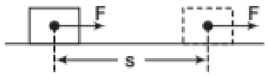
(ii) The point of application of the force moves in the direction of the force.

e.g. Work is done when an engine pulls a train, a man goes up a hill, a horse pulls a cart, etc.

Work Done by a Constant Force

Work done by the force (constant force) is the product of component of force in the direction of the displacement and the magnitude of the displacement.

If a constant force **F** is applied on a body and the body has a displacement **s** in the direction of the force as shown in figure.



Work done, when force and displacement are in the same direction

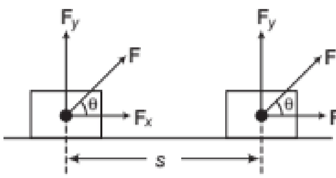
Then, the work done on the body by the force is given by

$$\boxed{\text{Work done, } W = \mathbf{F} \cdot \mathbf{s}}$$

Thus, work done by a force is the **dot product** of force and displacement.

Work Done When Force and Displacement are Inclined to Each Other

Sometimes the displacement **s** is not in the direction of **F** as shown in figure.



Work done when force and displacement are inclined to each other

In such a case, we find the work done by resolving **F** into two rectangular components.

(i) $\mathbf{F}_x$ along the direction of displacement **s**, such that

$$|\mathbf{F}_x| = F\cos\theta$$

(ii) $\mathbf{F}_y$ along perpendicular to displacement **s**, such that

$$|\mathbf{F}_y| = F\sin\theta$$

The component $\mathbf{F}_y$ does not work as the body does not move up or down. All the work is done by the component $\mathbf{F}_x$.

$$\text{Work done } (W) = |\mathbf{F}| \cdot |\mathbf{s}| = (F\cos\theta) \cdot s$$

$$\boxed{\text{Work done, } W = Fs\cos\theta}$$

Thus, work done is the dot product of force and displacement vector. Hence, work is a scalar quantity.

Two cases can be considered for the maximum and minimum work.

Case I When F and s are in the same direction, i.e. $\theta = 0°$, then work done is

$$W = Fs\cos 0° = Fs(1) = Fs$$

i.e. maximum work done by the force.

Case II When F and s are perpendicular to each other, then

$$W = \mathbf{F} \cdot \mathbf{s} = Fs\cos 90° = Fs(0) = 0$$

i.e. no work done by the force, when a body moves in a direction perpendicular to the force.

Note

For a particular displacement, work done by a force is independent of type of motion i.e. whether it moves with constant velocity, constant acceleration or retardation.

Dimensions and Units of Work

As, work = force × distance $= [MLT^{-2}] \times [L]$

$$W = [ML^2T^{-2}]$$

This is the dimensional formula of work.

The units of work are of two types:

1. Absolute units 2. Gravitational units

1. Absolute Units

Work done is said to be one absolute unit, if an absolute unit of force displaces a body through a unit distance in the direction of the force.

(i) **Joule** It is the absolute unit of work in SI, named after British physicist James Prescott Joule (1811 – 1869).

One joule of work is said to be done when a force of one newton displaces a body through a distance of one metre in its own direction.

From work done $(W) = Fs\cos\theta$,

$$1 \text{ J} = 1 \text{ N} \times 1 \text{ m} \times \cos 0° = 1 \text{ N-m}$$

(ii) **Erg** It is the absolute unit of work in CGS system. One erg of work is said to be done if a force of one dyne displaces a body through a distance of one centimetre in its own direction.

From work done $(W) = Fs\cos\theta$

$$1 \text{ erg} = 1 \text{ dyne} \times 1 \text{ cm} \times \cos 0° = 1 \text{ dyne-cm}$$

Relation between Joule and Erg

$$1 \text{ J} = 1 \text{ N} \times 1 \text{ m}$$
$$= 10^5 \text{ dyne} \times 10^2 \text{ cm} = 10^7 \text{ dyne-cm}$$

or $\boxed{1 \text{ J} = 10^7 \text{ erg}}$

2. Gravitational Units

Work is said to be one gravitational unit if a gravitational unit of force displaces a body through a unit distance in the direction of the force.

(i) **Kilogram-metre** (kg-m) It is the gravitational unit of work in SI system. One kilogram-metre of work is said to be done when a force of one kilogram weight displaces a body through a distance of one metre in its own direction.

We know work done $(W) = Fs\cos\theta$

$$1 \text{ kg-m} = 1 \text{ kg wt} \times \cos 0°$$
$$= 9.8 \text{ N} \times 1\text{m} \times 1 = 9.8 \text{ J}$$

i.e. $1 \text{ kg-m} = 9.8 \text{ J}$

(ii) **Gram-centimetre** (g-cm) It is the gravitational unit of work in CGS system. One gram-centimetre of work is said to be done when a force of one gram-centimetre weight displaces a body through a distance of one centimetre in its own direction.

∴ $1 \text{ g-cm} = 1 \text{ g-wt} \times 1 \text{ cm} = 980 \text{ dyne} \times 1 \text{ cm}$

or $1 \text{ g-cm} = 980 \text{ erg}$

EXAMPLE |1| A Lawn Roller

A lawn roller has been pushed by a gardener through a distance of 30 m. What will be the work done by him if he applies a force of 30 kg-wt in the direction inclined at 60° to the ground? Take, $g = 10 \text{ m/s}^2$

Sol. Given, Displacement $s = 30$ m

Force, $F = 30 \text{ kg-wt} = 30 \times 10 = 300 \text{N}$

Angle between force and ground, $\theta = 60°$

The work done by the gardener,

$$W = F \cdot s = Fs \cos\theta = 300 \times 30 \times \cos 60°$$

$$W = 4500 \text{ J}$$

EXAMPLE |2| A Coolie

A coolie is holding a bag by applying a force of 15 N. He moves forward and covers the horizontal distance of 8 m and then he climbs up and covers the vertical distance of 10 m. What will be the work done by him?

The net work done by coolie is the sum of work done to cover the horizontal direction and the work done to climb up in the vertical direction.

Sol. Given, $F = 15 \text{ N}, s_1 = 8 \text{ m}$ and $s_2 = 10 \text{m}$

As coolie is walking horizontally, therefore, the angle between the bag and distance covered is 90°.

∴ Work done to cover the distance of 8 m.

$$W_1 = Fs_1 \cos\theta$$

$$= 15 \times 8 \cos 90° = 0 \text{ J}$$

When the coolie climb.

Thus, $\theta = 0°$

The work done, $W_2 = Fs_2 \cos\theta$

$$= 15 \times 10 \times \cos 0° = 150 \text{ J}$$

The net work done by him

$$W = W_1 + W_2 = 0 + 150 = 150 \text{ J}$$

Nature of Work Done in Different Situations

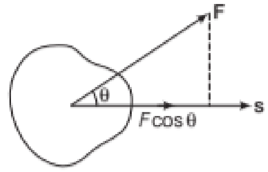
Although work done is a scalar quantity, its value may be positive, negative or even zero, as discussed below:

1. Positive Work

If a force acting on a body has a component in the direction of the displacement, then the work done by the force is positive.

As shown in figure, when θ is acute, then $\cos\theta$ is positive i.e. $0 \le \theta < 90°$.

∴ Work done $(W) = Fs\cos\theta$ = positive value $[0 \le \theta < 90°]$



Positive work done ($\theta < 90°$)

e.g.

(i) When a horse pulls a cart, the applied force and the displacement are in the same direction. So, work done by horse is positive.

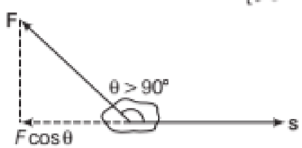
(ii) When a load is lifted, then the lifting force and displacement act in the same direction. So, work done by the lifting force is positive.

(iii) When a spring is stretched, work done by the stretching force is positive.

2. Negative Work

If a force acting on the body has a component in the opposite direction of displacement, then work done is negative. As shown in figure, when θ is obtuse, then $\cos\theta$ is negative.

Work done $(W) = Fs\cos\theta$ = negative value $[90° < \theta < 180°]$



Negative work done ($\theta > 90°$)

e.g.

(i) When brakes are applied to a moving vehicle, the work done by the braking force is negative, this is because the braking force and the displacement act in opposite directions.

(ii) When a body is dragged along a rough surface, then work done by the frictional force is negative. This is because the frictional force acts in a direction opposite to that of the displacement.

(iii) When a body is lifted, then work done by gravitational force is negative. This is because the gravitational force acts vertically downwards while the displacement is in the vertically upward direction.

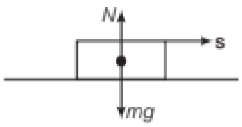
3. Zero Work

Work done by force is zero, if the body gets displaced in the direction perpendicular to the direction of the applied force.

Work done $(W) = Fs\cos 90° = Fs \cdot 0$ = zero e.g.

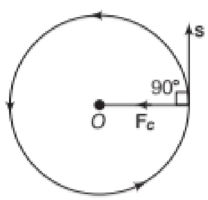
(i) Consider a body sliding over a horizontal surface. The work done by the force of gravity and the normal reaction of the surface will be zero.

This is because both the forces of gravity and reaction act normally to the displacement as shown in figure.



Work done by force of gravity and normal force

(ii) Consider a body moving in a circle with constant speed. At every point of the circular path, the centripetal force and the displacement are mutually perpendicular as shown in figure.

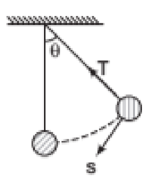


Work done by centripetal force

So, the work done by the centripetal force is zero. The same argument can be applied to a satellite moving in a circular orbit.

In this case, the gravitational force is always perpendicular to displacement. So, work done by gravitational force is zero.

(iii) As shown in figure, the tension in the string of simple pendulum is always perpendicular to displacement. So, work done by the tension is zero.



Work done by tension

EXAMPLE |3| Leaving Skid Marks

A cyclist comes to a skidding stop in 10 m. During this process, the force on the cycle due to the road is 200 N and is directly opposite to the motion. (i) How much work does the road do on the cycle? (ii) How much work does the cycle do on the road? **[NCERT]**

Work done on the cycle by the road is the work done by the stopping force of friction on the cycle due to the road.

Sol. (i) The stopping force and the displacement make an angle of $180°$ with each other. Thus, work done by the road or the work done by the stopping force is

$$W_r = Fs\cos\theta$$
$$= 200 \times 10 \times \cos 180° = -2000 \text{ J}$$

Negative sign shows that work done by road-on cycle.

(ii) According to Newton's third law, an equal and opposite force acts on the road due to the cycle. Its magnitude is 200 N. However, the road undergoes no displacement. So, work done by the cycle on the road is zero.

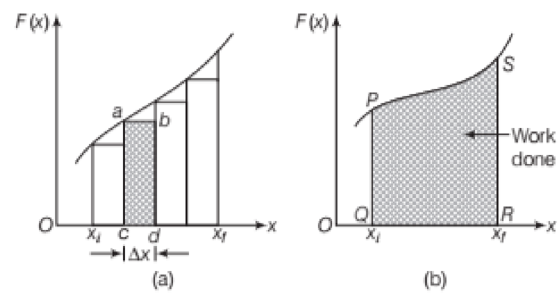
Work Done by a Variable Force

Let a variable force F acts on a body along the fixed direction, say x-axis. The magnitude of the force F depends on x, as shown by force-displacement graph in figure.

Let us calculate the work done when the body moves from the initial position x_i to the final position x_f under a force $F(x)$.

The displacement can be divided into a large number of small equal displacements, i.e. Δx. During small displacement Δx, the force F can be assumed to be constant. Then, the work done is

$$\Delta W = F(x)\Delta x = \text{ Area of rectangle } abcd.$$



Calculation of work done by a variable force

Adding areas of all the rectangles in figure (a), we get the total work done as

$$W \equiv \sum_{x_i}^{x_f} F(x)\,\Delta x$$

If the displacements are much small (i.e. are allowed to approach zero), then the number of terms in the sum increases without limit, but the sum approaches to definite value equal to the area under the curve. Thus, the total work done is

$$W = \lim_{\Delta x \to 0} \sum_{x_i}^{x_f} F(x)\,\Delta x = \int_{x_i}^{x_f} F(x)\,dx$$

$$W_{x_i \to x_f} = \int_{x_i}^{x_f} \mathbf{F} \cdot d\mathbf{x} = \int_{x_i}^{x_f} (F\cos\theta) \cdot dx$$

$$= \text{Area of } PQRS$$

$$W_{x_i \to x_f} = \text{Area under the force-displacement curve}$$

Hence, for a varying force the work done is equal to the definite integral of the force over the given displacement as shown in figure.

When the magnitude and direction of a force vary in three dimensions. So, it can be expressed in terms of rectangular components, we get

Force, $\mathbf{F} = F_x \hat{\mathbf{i}} + F_y \hat{\mathbf{j}} + F_z \hat{\mathbf{k}}$ and

Displacement, $\mathbf{dx} = dx\,\hat{\mathbf{i}} + dy\,\hat{\mathbf{j}} + dz\,\hat{\mathbf{k}}$

So, work done from x_i to x_f

$$\text{Work done, } W = \int_{x_i}^{x_f} F_x dx + \int_{x_i}^{x_f} F_y dy + \int_{x_i}^{x_f} F_z dz$$

where, F_x, F_y and F_z are the rectangular components of force in x, y and z-directions, respectively.

Note

When the body is in equilibrium (either static or dynamic), then resultant force is zero. Therefore, work done $W = 0$.

EXAMPLE |4| Work Done in Moving the Particles

Force $\mathbf{F} = [3x^2\hat{\mathbf{i}} + 4\hat{\mathbf{j}}]$ N with x in metres, acts on a particle. How much work is done on the particles as it moves from coordinates (2m, 3m) to (3m, 0m)?

Sol. Given, force $F = 3x^2\,\hat{\mathbf{i}} + 4\,\hat{\mathbf{j}}$

Coordinates $= (\underset{x_i}{2\text{m}}, \underset{y_i}{3\text{m}})$ to $(\underset{x_f}{3\text{m}}, \underset{y_f}{0\text{m}})$

Here, the force is variable force. Thus, work done is

$$W = \int_{x_i}^{x_f} F_x \cdot dx + \int_{y_i}^{y_f} F_y dy$$

$$= \int_2^3 3x^2\, dx + \int_3^0 4\, dy = 3\left[\frac{x^3}{3}\right]_2^3 + 4\,[y]_3^0$$

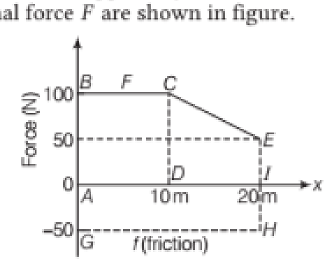
$$= \frac{3}{3} \times [27 - 8] + 4[0 - 3] = 19 - 12$$

$$W = 7.0 \text{ J}$$

EXAMPLE |5| Not Easy to Push a Trunk

A woman pushes a trunk on railway platform which has a rough surface. She applies a force of 100 N over a distance of 10 m. Thereafter, she gets progressively tired and her applied force reduces linearly with distance to 50 N. The total distance by which the trunk has been moved is 20 m. Plot the force applied by the woman and the frictional force, which is 50 N. Calculate the work done by the two forces over 20 m. **[NCERT]**

Sol. Plots of force F applied by the woman and the opposing frictional force F are shown in figure.



Clearly, at $x = 20$ m, $F = 50$ N

As we know the force of friction f opposes the applied force F, so it has been shown on the negative side of the force-axis.

Work done by the force F applied by the woman

W_F = Area of rectangle $ABCD$ + Area of trapezium $CEID$

$$= 100 \times 10 + \frac{1}{2}(100 + 50) \times 10 = 1000 + 750$$

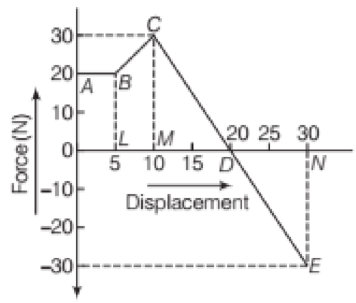
$$= 1750 \text{ J}$$

Work done by the frictional force,

$$W_f = \text{Area of rectangle } AGHI = (-50) \times 20 = -1000 \text{ J}$$

EXAMPLE |6| Work Done by Man using Variable Force

The force applied by a man in pushing a block varies with displacement as shown in figure. If the force is expressed in Newton and displacement in metres, find the work done by him.



Sol. Given, displacement and forces are

At point B, $d_1 = 5$ m, $f_1 = 20$ N

At point C, $d_2 = 10$ m, $f_2 = 30$ N,

At point D, $d_3 = 20$ m, $f_3 = 0$ N,

At point E, $d_4 = 30$ m, $f_4 = -30$ N

Work done by the man = ?

Area under the rectangle $OABL$ is equal to work done W_{OABL}.

$$W_{OABL} = 20 \times 5 = 100 \text{ J}$$

[$\because$ Area = length × breadth]

Area under the trapezium $BCML$ is equal to work done W_{BCML}.

$$W_{BCML} = \frac{1}{2} \times [20 + 30] \times 5$$

$$\left[\because \text{ area} = \frac{1}{2} \times (\text{sum of parallel side}) \times \text{height}\right]$$

$$W_{BCML} = \frac{1}{2} \times 50 \times 5 = 125 \text{ J}$$

Area under the $\Delta\, CDM$ is equal to work done W_{CDM}.

$$W_{CDM} = \frac{1}{2} \times 10 \times 30 = 150 \text{ J}$$

$$\left[\because \text{ area} = \frac{1}{2} \times \text{base} \times \text{height}\right]$$

Area under the Δ *DEN* is equal to work done W_{DEN}.

$$W_{DEN} = \frac{1}{2} \times (10) \times -30 = -150 \text{ J}$$

$$\left[\because \text{area} = \frac{1}{2} \times \text{base} \times \text{height}\right]$$

Total amount of work done

$$= 100 \text{ J} + 125 \text{ J} + 150 \text{ J} - 150 \text{ J} = 225 \text{ J}$$

CONSERVATIVE FORCE

If the work done by the force in displacing an object depends only on the initial and final positions of the object and not on the nature of the path followed between the initial and final positions, such a force is known as **conservative force**.

e.g. **Gravitational force** is a conservative force.

Work Done by Conservative Forces

(i) Work done by or against a conservative force, in moving a body from one position to another, depends only on the initial and final positions of the body.

(ii) It does not depend upon the nature of the path followed by the body in going from initial position to the final position.

(iii) Work done by or against a force is said to be conservative, if the work done by the force in moving a particle along a closed path (round trip), net work done is zero.

Net Work Done by a Body in Moving Over the Round Trip

In case of gravitational force, if work done in moving the body from position *P* to *Q*, against the gravity (g $\rightarrow$ −Ve), then work done in moving the body from position *Q* to *P*, by gravity, as taken be positive. i.e.

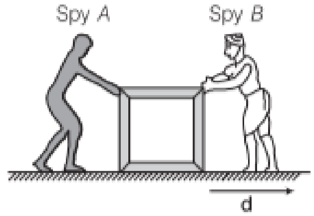
$$W_{PQ} = -W_{QP}, W_{PQ} + W_{QP} = 0$$

Thus, net work done by a body in moving over the round trip ($P \rightarrow Q \rightarrow P$) is zero.

EXAMPLE |7| Sliding Object

Figure shows two industrial spies sliding an initially stationary 250 kg floor safe, a displacement **d** of magnitude 10 m, straight towards their truck. The push $\mathbf{F}_1$ of spy *A* is 15 N directed at an angle of 30° downward from the horizontal, the pull $\mathbf{F}_2$ of spy *B* is 12 N directed at 40° above the horizontal plane.

The magnitudes and direction of these forces do not change as the safe moves, and the floor and safe make frictionless contact. [tan 40° = 0.8390, sin 40° = 0.6427]

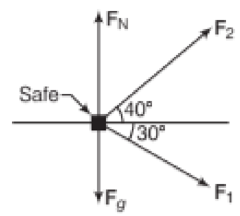


(i) What is the net work done on the safe by forces $\mathbf{F}_1$ and $\mathbf{F}_2$ during the displacement **d** ?

(ii) During the displacement, what is the work W_g done on the safe by the gravitational force $\mathbf{F}_g$ and what is the work W_N done on the safe by normal $\mathbf{F}_N$ from the floor?

The net work done on the safe by forces $\mathbf{F}_1$ and $\mathbf{F}_2$ is the sum of works they do individually. Because we can treat the safe as a particle and the forces are constant in both magnitude and direction, we can use either ($W = Fd \cos \phi$) or ($W = \mathbf{F} \cdot \mathbf{d}$) to calculate those works. Since, we know the magnitudes and directions of the forces, we choose $W = Fd \cos \phi$. First draw the FBD of the safe.

Sol. (i)



For first spy, $F_1 = 15$ N, $d = 10$ m and $\phi_1 = 30°$

Thus, work done

$$W_1 = F_1 d \cos \phi_1,$$

$$W_1 = 15 \times 10 \cos 30° = 130 \text{ J}$$

For Second spy, $F_2 = 12$ N, $d = 10$ m and $\phi_2 = 40°$

Thus, work done

$$W_2 = F_2 d \cos \phi_2$$

$$W_2 = 12 \times 10 \times \cos 40° \quad \left[\because \cos 40° = \frac{\sin 40°}{\tan 40°}\right]$$

$$= 12 \times 10 \times 0.77 \equiv 92 \text{ J}$$

Hence, net work done $W = W_1 + W_2 = 130 + 92 = 222$ J

(ii) These two forces (i.e. F_1 and F_2) are constant in both magnitude and direction, we can find out the work done on the safe by the gravitational force.

$$W_g = F_g d \cos 90°$$

Take $\phi = 90°$ because the direction of the gravitational force is perpendicular to the displacement of safe.

$$W_g = mgd \cos 90° = 0$$

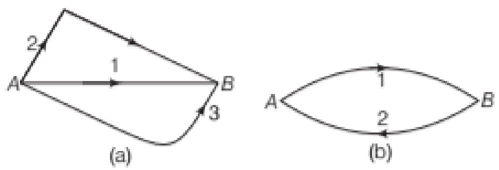
and
$$W_N = F_N d \cos 90°$$
$$= F_N d \cdot 0 = 0$$

$\therefore$ These forces are perpendicular to the displacement of safe, they do zero work.

NON-CONSERVATIVE FORCE

If work done by a force in displacing an object from one position to another, depends upon the path between the two positions. Such a force is known as **non-conservative force**.

Let W_1, W_2 and W_3 denote the net work done in moving a body from A to B along three different path 1, 2 and 3 respectively as shown in figure. If the force is non-conservative, then $W_1 \neq W_2 \neq W_3$



If Fig. (b), a particle is moving in a closed path $A \to 1 \to B \to 2 \to A$. If W_1 is work done in moving the particle from $A \to 1 \to B$, and W_2 is work done in moving the particle from $B \to 2 \to A$, then for a non-conservative force $|W_1| \neq |W_2|$

Hence, net work done along the closed path, $A \to B \to A$ is not zero.

The common examples of non-conservative forces are

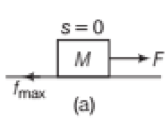
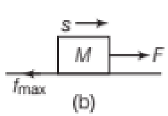
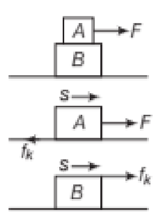
(i) Force of friction (ii) Viscous force

(iii) Tension

Work Done by Non-conservative Forces (Friction Forces)

As we know that friction force is always opposite to the relative motion so that the work done by friction may be positive, zero and negative.

Work Done due to Friction Forces

Case I	Case II	Case III
Fig. (a) shows a situation where a block is pulled by a force F which is insufficient to overcome the friction f_{max} i.e. $F < f_{max}$	Fig. (b) shows a situation where a block is pulled by a force F which is large to overcome friction f_{max} i.e. $F > f_{max}$. Here, the work done by friction force is negative.	Fig. (c) shows that block A is placed on block B and when block A is pulled with a force F, friction force does positive work on block B and negative work on block A.
 (a)	 (b)	 (c)
$F < f_{max}$ $F < \mu_s N$ $F < \mu_s Mg$ $\therefore$ Work done, $W = 0$	$F > f_{max}$ $f_{max} = Mg$ $W = -\mu_k Mgs$	Work done $W_{Friction}$ = – ve for A. Work done $W_{Friction}$ = + ve for B.

EXAMPLE |8| Sliding of Wooden Block on Inclined Plane

A wooden block of mass 1 kg is pushed up a surface inclined to horizontal at angle of 30° by a force of 10 N parallel to the inclined surface [figure]. The coefficient of friction between block and the incline is 0.1. If the block is pushed up by 10 m along the incline. Calculate.

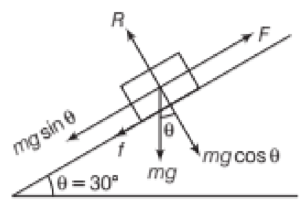
(i) Work done against gravity.

(ii) Work done against force of friction.

(iii) Acceleration of block when it moves upward.

Sol. Given, $m = 1$ kg, $\theta = 30°$, $F = 10$ N, $\mu = 0.1$, $s = 10$ m,

$W_g = ?$, $W_f = ?$, $a = ?$



Work done against gravity,

$$W_g = mg \sin\theta \times s$$
$$W_g = mg \sin\theta \times s$$
$$= 1 \times 10 \sin 30° \times 10$$
$$= \frac{100}{2} = 50 \text{ J}$$

Work done against friction,

i.e. $W_f = F \times s$

$$W_f = F \times s = \mu R \times s$$
$$= \mu mg \cos\theta \times s$$
$$= 0.1 \times 1 \times 10 \cos 30° \times 10$$
$$= 10 \times 0.866 = 8.66 \text{ J}$$

Acceleration of block up the inclined plane, i.e.

$$F = ma + (mg \sin\theta + \mu\, mg \cos\theta)$$

$$\Rightarrow \quad a = \frac{F - (mg \sin\theta + \mu\, mg \cos\theta)}{m}$$

$$a = \frac{10 - \left(1 \times 10 \times \frac{1}{2} + 0.1 \times 10 \times 0.866\right)}{1}$$

$$= 4.13 \text{ m/s}^2$$

TOPIC PRACTICE 1

OBJECTIVE Type Questions

1. In which case, work done will be zero
(a) a weight-lifter while holding a weight of 100 kg on his shoulders for 1 min
(b) a locomotive against gravity when it is running on a level plane with a speed of 60 kmh^{-1}
(c) a person holding a suitcase on his head and standing at a bus terminal
(d) All of the above

Sol. (d) Work done by weight-lifter is zero because there is no displacement. In a locomotive, work done is zero because force and displacement are mutually perpendicular to each other.

While a person holding a suitcase, work done is zero because there is no displacement.

2. If the force and displacement of particle in the direction of force are doubled, then work done would be

(a) double (b) 4 times
(c) half (d) $\frac{1}{4}$ times

Sol. (b) $\because$ Work = Force × Displacement ...(i)
So, if the force and displacement of particle in the direction of force are doubled, then as per Eq. (i), their product will make the work done 4 times more than its initial value.

3. An electron and a proton are moving under the influence of mutual forces. In calculating the change in the kinetic energy of the system during motion, one ignores the magnetic force of one on another. This is, because [NCERT Exemplar]
(a) the two magnetic forces are equal and opposite, so they produce no net effect
(b) the magnetic forces do not work on each particle
(c) the magnetic forces do equal and opposite (but non-zero) work on each particle
(d) the magnetic forces are necessarily negligible

Sol. (b) When electron and proton are moving under influence of their mutual forces, the magnetic forces will be perpendicular to their motion, hence no work is done by these forces.

4. A particle is pushed by forces $2\hat{i} + 3\hat{j} - 2\hat{k}$ and $5\hat{i} - \hat{j} - 3\hat{k}$ simultaneously and it is displaced from point $\hat{i} + \hat{j} + \hat{k}$ to point $2\hat{i} - \hat{j} + 3\hat{k}$. The work done is

(a) 7 units (b) −7 units
(c) 10 units (d) −10 units

Sol. (b) Net force, $\mathbf{F} = 2\hat{i} + 3\hat{j} - 2\hat{k} + 5\hat{i} - \hat{j} - 3\hat{k}$
$$= 7\hat{i} + 2\hat{j} - 5\hat{k}$$
Displacement, $\mathbf{d} = 2\hat{i} - \hat{j} + 3\hat{k} - \hat{i} - \hat{j} - \hat{k} = \hat{i} - 2\hat{j} + 2\hat{k}$
Work done $= \mathbf{F}.\mathbf{d} = (7\hat{i} + 2\hat{j} - 5\hat{k}).(\hat{i} - 2\hat{j} + 2\hat{k})$
$$= 7 - 4 - 10 = -7 \text{ units}$$

5. A body is moving along a circular path. How much work is done by the centripetal force?
(a) −1 J (b) − 2 J (c) 3 J (d) Zero

Sol. (d) For a body moving along a circular path, the centripetal force acts along the radius while the displacement is tangential, i.e. $\theta = 90°$, therefore, $W = Fs\cos 90° = 0$.

VERY SHORT ANSWER Type Questions

6. Is work done a scalar or a vector?

Sol. Work done by a force for a certain displacement is a scalar quantity.

7. Under what condition is the work done by a force is zero inspite of displacement being taking place?

Sol. Work done by a force is zero inspite of displacement being taking place, if displacement is in a direction perpendicular to that of force applied.

8. Can acceleration be produced without doing any work? Give example.

Sol. Yes, for uniform circular motion, no work done but a centripetal acceleration is present.

9. In a game of tug of war, one team is slowly giving way to the other. Which team is doing positive work and which team negative?

Sol. The winning team (i.e. the team which is pulling the other team towards itself) is doing positive work and the losing team (i.e. the team slowly giving way to the other) is doing negative work.

10. Does the amount of work done depend upon the fact that how fast is a load raised or moved in the direction of force?

Sol. The amount of work does not depend upon the fact that how fast is a load raised or moved in the direction of force.

11. A body is moving along a circular path. How much work is done by the centripetal force?

Sol. For a body moving along a circular path, the centripetal force acts along the radius while the displacement is tangential, i.e. $\theta = 90°$, therefore, $W = Fs\cos 90° = 0$.

SHORT ANSWER Type Questions

12. A body constrained to move along the Z-axis of a coordinate system is subject to a constant force $\mathbf{F}$ given by $\mathbf{F} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N, where $\hat{\mathbf{i}}, \hat{\mathbf{j}}, \hat{\mathbf{k}}$ are unit vectors along the x, y and z-axes of the system, respectively. What is the work done by this force in moving the body a distance of 4 m along the z-axis? **[NCERT]**

Sol. Here, $\mathbf{F} = (-\hat{\mathbf{i}} + 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$ N and $\mathbf{s} = (4\hat{\mathbf{k}})$ m

$$W = \mathbf{F} \cdot \mathbf{s} = \mathbf{F}_z \cdot \mathbf{s}_z = 3 \times 4 = 12 \text{ N-m} = 12 \text{ J}$$

13. A trolley of mass 300 kg carrying a sand bag of 25 kg is moving uniformly with a speed of 27 km/h on a frictionless track. After a while, sand starts leaking out of a hole on the floor of the trolley at the rate of 0.05 kgs^{-1}. What is the speed of the trolley after the entire sand bag is empty? **[NCERT]**

Sol. In present problem, there are no net external forces. The sand is slowly leaking downwards from the hole. Consequently, normal reaction acting vertically upwards also slowly decreases.

But it does not do any work because it is at right angle to trolley's motion. Thus, there is no change in speed of the trolley and it will continue among moving with a uniform speed of 27 kmh^{-1}.

14. Find the work done in pulling and pushing a roller through 100 m horizontally when a force of 1500 N is acting along a chain making an angle of 60° with ground. Assume the floor to be smooth.

Sol. Here, force $F = 1500$ N

and displacement, $s = 100$ m, $\theta = 60°$

$\therefore$ Work done, $W = Fs\cos\theta = 1500 \times 100 \times \cos 60°$

$$= 1500 \times 100 \times \frac{1}{2} = 75000 \text{ J} = 75 \text{ kJ}$$

15. Calculate the work done by a car against gravity in moving along a straight horizontal road. The mass of the car is 400 kg and the distance moved is 2 m. **[NCERT Exemplar]**

Sol. The work done by a car against gravity is zero because force of gravity is vertical and motion of car is along a straight horizontal road.

As angle θ between directions of force and displacement is 90°, hence work done is zero.

LONG ANSWER Type I Questions

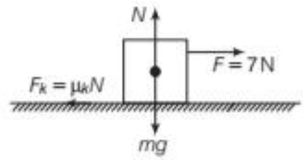
16. A body of mass of 2 kg initially at rest moves under the action of an applied horizontally force of 7 N on a table with coefficient of kinetic friction = 0.1. Compute the **[NCERT]**

(i) work done by the applied force in 10 s,

(ii) work done by friction in 10 s,

(iii) work done by the net force on the body in 10 s,

Sol. Here, applied force $F = 7$ N



and opposing friction force, $f = \mu_k \cdot N = \mu_k \cdot mg$

$$= 0.1 \times 2 \times 9.8 = 1.96 \text{ N}$$

$\because$ Net accelerating force $= F - f = 7 - 1.96 = 5.04$ N

$\therefore$ Acceleration, $a = \frac{\text{force}}{\text{mass}} = \frac{5.04 \text{ N}}{2 \text{ kg}} = 2.52 \text{ ms}^{-2}$

(i) Distance covered in 10 s (assuming $u = 0$)

$$s = 0 + \frac{1}{2}at^2 = \frac{1}{2} \times 2.52 \times (10)^2 = 126 \text{ m}$$

$\therefore$ Work done by the applied force

$$W = Fs = 7 \times 126 = +882 \text{ J}$$

(ii) Work done by friction in 10 s

$$W' = fs\cos 180° = -1.96 \times 126 = -247 \text{ J}$$

(iii) Work done by net force in

$$10 \text{ s} = W - W' = 882 - 247 = +635 \text{ J}$$

17. A body of mass 3 kg is under a constant force, which causes a displacement S in metre in it, given by the relation $S = \frac{1}{3}t^2$, where t is in second. Find the work done by the force in 2s.

Sol. Work done by the force = force × displacement

or $\quad W = F \times S \quad$ ...(i)

But from Newton's 2nd law, we have

Force = mass × acceleration

i.e. $\quad F = ma \quad$ ...(ii)

Hence, from Eqs. (i) and (ii), we get

$$W = m\left(\frac{d^2s}{dt^2}\right)S \quad \left[\because a = \frac{d^2s}{dt^2}\right] \quad \text{...(iii)}$$

Now, we have $S = \frac{1}{3}t^2$

$$\therefore \quad \frac{d^2s}{dt^2} = \frac{d}{dt}\left[\frac{d}{dt}\left(\frac{1}{3}t^2\right)\right] = \frac{d}{dt} \times \left(\frac{2}{3}t\right) = \frac{2}{3}\frac{dt}{dt} = \frac{2}{3}$$

Hence Eq. (iii) becomes

$$W = \frac{2}{3}ms = \frac{2}{3} \times m \times \frac{1}{3}t^2 = \frac{2}{9}mt^2$$

We have, $m = 3$ kg, $t = 2s$

$$\therefore \quad W = \frac{2}{9} \times 3 \times (2)^2 = \frac{8}{3} \text{J}$$

18. A boy has a bag of sand of mass 20 kg. First of all, he keeps the bag on his head and moves 10 m. Second time, he drags the bag through 10 m on a frictionless surface with coefficient of friction $\mu = 0.1$. In which case, he does more work?

Sol. We know, work done is given by $W = Fs \cos \theta$

In first case, angle (θ) between F and s is 90° because weight of the bag, i.e. force acts perpendicular to the displacement.

$\therefore \quad W = Fs \cos 90° = 0$

In second case, F and s are in the same direction, so $\theta = 0°$.

Here, $F = \mu mg = 0.1 \times 20 \times 9.8 = 19.6$ N

$\therefore \quad W = 19.6 \times 10 \times \cos 0 = 196$ J

So, work done in second case is more.

19. A man weighing 50 kg supports a body of 25 kg on his head. What is the work done when he moves a distance of 20 m up an incline of 1 in 10? Take, $g = 9.8 \text{ m/s}^2$.

Sol. Here, mass of body $= 50 + 25 = 75$ kg

$$\sin \theta = \frac{1}{10}$$

Distance, $s = 20$ m, $g = 9.8 \text{ m/s}^2$

Force needed to be applied against gravity

i.e. $F = mg \sin \theta$

Work done, $W = Fs = mg \sin \theta \times s$

$$= 75 \times 9.8 \times \frac{1}{10} \times 20$$

$$= 1470 \text{ J}$$

20. A particle moves along the x-axis from $x = 0$ to $x = 5$m under the influence of a force given by $f(x) = 7 - 2x + 3x^2$. Calculate the work done.

Sol. As work done, $dW = Fdx$

$$W = \int_0^5 F \cdot dx = \int_0^5 (7 - 2x + 3x^2)dx$$

$$= \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3}\right]_0^5$$

$$= 7(5-0) - (5^2 - 0) + (5^3 - 0) = 135 \text{ J}$$

LONG ANSWER Type II Questions

21. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative.

(i) Work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.

(ii) Work done by gravitational force in the above case.

(iii) Work done by friction on a body sliding down on inclined plane.

(iv) Work done by an applied force on a body moving on a rough horizontal plane with uniform velocity.

(v) Work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

[NCERT]

Sol. (i) Work done by the man against the force of gravity while lifting a bucket out of a well by means of a rope tied to the bucket is +ve, because force and displacement are in same direction.

(ii) Work done by gravitational force in the above case is −ve because direction of gravitational force is opposite to the displacement.

(iii) Work done is −ve because direction of friction force is opposite to sliding motion.

(iv) Work done by an applied force on a body moving on a rough horizontal plane is positive because force is being applied in the direction of motion so as to overcome friction.

(v) Work done is negative because the resistive force of air acts in a direction opposite to the direction of motion of the vibrating pendulum.

22. Calculate work done in raising a stone of mass 5 kg of specific gravity 3 immersed in water from a depth of 6 m to 1 m below surface of water. (Take, $g = 10 \text{ ms}^{-2}$)

Sol. As the stone weight acts downwards and upthrust acts upwards.

Thus, net weight of stone can be calculated.

$$\text{Specific gravity} = \text{Relative density} = \frac{\text{Density of stone}}{\text{Density of water}}$$

Thus, density of stone $= 3 \times 10^3 \text{ kg/m}^3$

$$\text{Volume of stone} = \frac{\text{Mass of stone}}{\text{Density of stone}}$$

$$= \frac{5}{3 \times 10^3} = \frac{5}{3} \times 10^{-3} \text{ m}^3$$

Upthrust on stone = Weight of liquid displaced

$$= \frac{5}{3} \times 10^{-3} \times \rho_{water} \times g$$

$$= \frac{5}{3} \times 10^{-3} \times 10^3 \times 10 = \frac{50}{3} \text{ N}$$

Thus, net weight of stone, $mg' = mg$ − upthrust

$$mg' = \left(5 \times 10 - \frac{50}{3}\right) = \frac{100}{3} \text{ N}$$

Now, force required to raise the stone, $F = \frac{100}{3}$ N

Work required to raise the stone,

$$W = Fd = \frac{100}{3} \times 5 = \frac{500}{3} \text{ J} = 166.6 \text{ J}$$

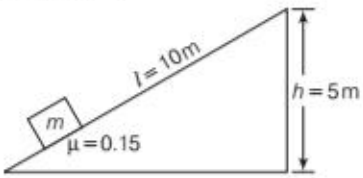
23. A body of mass 0.3 kg is taken up an inclined plane length 10 m and height 5 m and then allowed to slide down the bottom again. The coefficient of friction between the body and the plane is 0.15. What is the

(i) work done by gravitational force over the round trip?

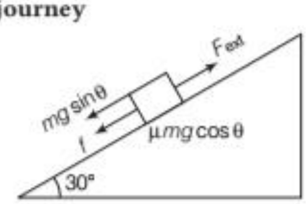
(ii) work done by the applied force over the upward journey?

(iii) work done by the frictional force over the round trip?

(iv) kinetic energy of the body at the end of trip? ($g = 10\ \text{ms}^{-2}$)



Sol. **Upward journey**



Let us calculate work done by different forces over upward journey.

Work by gravitational force

$$W_1 = (mg\sin\theta)s\cos 180°$$

$$W_1 = 0.3 \times 10 \sin 30° \times 10(-1)$$

$$W_1 = -15\ \text{J}$$

Work by force of friction

$$W_2 = (\mu\ mg\cos\theta)s\cos 180°$$

$$W_2 = 0.15 \times 0.3 \times 10\cos 30° \times 10[-1]$$

$$W_2 = -3.879\ \text{J}$$

Work done by external force

$$W_3 = F_{ext} \times s \times \cos 0°$$

$$W_3 = [mg\sin\theta + \mu\ mg\cos\theta] \times 10 \times 1$$

$$W_3 = 18.897\ \text{J}$$

Downward journey

$$mg\sin 30° > \mu\ mg\cos 30°$$

Work done by the gravitational force

$$W_4 = mg\sin 30° \times s\cos 0°$$

$$W_4 = 0.3 \times 10 \times \frac{1}{2} \times 10 = +15\ \text{J}$$

Work done by the frictional force

$$W_5 = \mu mg\cos 30° \times s\cos 180°$$

$$= 0.15 \times 0.3 \times \frac{10\sqrt{3}}{2} \times 10 \times (-1)$$

$$= -3.897\ \text{J}$$

(i) Work done by gravitational force over the round trip

$$= W_1 + W_4 = 0\ \text{J}$$

(ii) Work done by applied force over upward journey

$$= W_3 = 18.897\ \text{J}$$

(iii) Work done by frictional force over the round trip

$$W_2 + W_5 = -3.897 + (-3.897) = -7.794\ \text{J}$$

(iv) Kinetic energy of the body at the end of the trip

$$W_4 + W_5 = 11.103\ \text{J}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. A body of mass 0.5 kg travels in a straight line by applying a force, $F = \frac{3}{2}ma^2x^2$, where $a = 5\ \text{m}^{-1/2}\text{s}^{-1}$. The work done by the net force during its displacement from $x = 0$ to $x = 2$ m is **[NCERT Exemplar]**

(a) 1.5 J (b) 50 J (c) 10 J (d) 100 J

2. Work done by gravitational force in one revolution of the earth around the sun on its elliptical path is zero because

(a) force is always perpendicular to displacement
(b) displacement is zero
(c) displacement is positive
(d) displacement is negative

3. Work done by a body against friction always results in

(a) loss in kinetic energy
(b) loss in potential energy
(c) gain in kinetic energy
(d) gain in potential energy

4. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is **[NCERT Exemplar]**

(a) + 2000J
(b) – 200J
(c) zero
(d) – 20, 000J

5. A force $\mathbf{F} = 5\hat{\mathbf{i}} + 6\hat{\mathbf{j}} - 4\hat{\mathbf{k}}$ acting on a body produces a displacement $\mathbf{s} = 6\hat{\mathbf{i}} + 5\hat{\mathbf{k}}$. The work done by the force is

(a) 18 units (b) 15 units
(c) 12 units (d) 10 units

Answer

1. (b)	2. (b)	3. (a)	4. (c)	5. (d)

VERY SHORT ANSWER Type Questions

6. Give the conditions under which a force is called conservative force.

7. A man raises a mass m to a height h and then shifts it horizontally by a length x. What is the work done against the force of gravity?

8. A mass is moving in a circular path with constant speed. What is the work done in $\frac{3}{4}$th of a rotation? [**Ans.** zero]

SHORT ANSWER Type Questions

9. What is meant by zero work? Give any one example.

10. Mountain roads wind up gradually instead of going straight up the slope. Why?

11. A man moves on a straight horizontal road with a block of a mass 2 kg in his hand. If he moves a distance of 40 m with acceleration of $0.5\ m/s^2$. Calculate work done by the man on the block during motion. [**Ans.** 40J]

LONG ANSWER Type I Questions

12. What is a conservative and non-conservative forces? Explain various properties.

13. Define the term work. Show that work done is equal to the dot product of force and displacement vectors.

14. What is meant by positive work, negative work and zero work? Give one example of each.

15. What is the amount of work done by
(i) a weight-lifter in holding a weight of 120 kg on his shoulder for 30 s and
(ii) a locomotive against gravity, if it is travelling on a level plane? [**Ans.** zero, zero]

LONG ANSWER Type II Questions

16. What is a conservative force? Prove that gravitational force is conservative, while frictional force is non-conservative.

17. Obtain mathematically and graphically the work done by a variable force.

18. A simple pendulum of length 1m has a wooden bob of mass 1 kg. It is struck by a bullet of mass 10^{-2} kg moving with a speed of 2×10^2 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back. [Take, $g = 10\ m/s^2$] [**Ans.** 0.2 m]

|TOPIC 2|

Energy

The energy of a body is defined as its capacity or ability for doing work.

(i) Like work, energy is a scalar quantity having magnitude only and no direction.

(ii) The dimensions of energy are the same as the dimensions of work i.e. $[M^1L^2T^{-2}]$.

(iii) It is measured in the same unit as work i.e. joule in SI and erg in CGS system.

Energy can exist in various forms such as mechanical energy (potential energy and kinetic energy), sound energy, heat energy, light energy, etc. Some practical units of energy and their equivalence to joule is given in the table below

Some Other Units of Work or Energy

S. No.	Unit	Symbol	Value in SI
1.	Erg	erg	10^{-7} J
2.	Electron volt	eV	1.6×10^{-19} J
3.	Calorie	cal	4.186 J
4.	Kilowatt hour	kWh	3.6×10^6 J

KINETIC ENERGY

The energy possessed by a body by virtue of its motion is called **kinetic energy**. In other words, the amount of work done, i.e. a moving object can do before coming to rest is equal to its kinetic energy.

$$\therefore \text{ Kinetic energy, KE} = \frac{1}{2}mv^2$$

where, m is a mass and v is the velocity of a body, e.g.

(i) A bullet fired from a gun can pierce a target due to its kinetic energy.

(ii) The kinetic energy of a fast stream of water is used to run water mills.

(iii) The kinetic energy of air is used to run wind mills.

(iv) The kinetic energy of a hammer is used in driving a nail into a piece of wood.

The units and dimensions of KE are Joule (in SI) and $[ML^2T^{-2}]$ respectively.

Kinetic energy of a body is always positive. It can never be negative.

Kinetic energy of a body depends upon the frame of reference, e.g. KE of a person of mass m sitting in a train moving with velocity v is $\left[\frac{1}{2}mv^2\right]$ in the frame of earth and KE of the same person $= 0$, in the frame of the train.

EXAMPLE |1| A Ballistics Demonstration

In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms^{-1} on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet? **[NCERT]**

Sol. Here, $m = 50.0 \text{ g} = 0.05 \text{ kg}$, $u = 200 \text{ ms}^{-1}$

$$\text{Initial KE} = \frac{1}{2}mu^2 = \frac{1}{2} \times 0.05 \times (200)^2 = 1000 \text{ J}$$

$$\text{Final KE} = 10\% \text{ of } 1000 \text{ J} = \frac{10 \times 1000}{100} = 100 \text{ J}$$

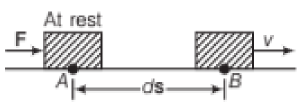
$$\text{or } \frac{1}{2}mv^2 = 100 \text{ J}$$

$$\therefore \quad v = \sqrt{\frac{2 \times 100}{m}} = \sqrt{\frac{2 \times 100}{0.05}} = 63.2 \text{ ms}^{-1}$$

Clearly, the speed reduces nearly by 68% and not by 90% by which the KE reduces.

Measurement of Kinetic Energy

Suppose a body is initially at rest and the force **F** is applied on the body to displace it through ds along the direction of the force. Then, the small work done as shown in figure.



A body displaced from A to B by applying force **F**

$$dW = \mathbf{F} \cdot d\mathbf{s} = Fds\cos 0° = Fds$$

According to Newton's second law of motion, we get

$$F = ma$$

where, a is acceleration produced on applying the force and m is the mass of the body.

$$\therefore \quad dW = mads = m\frac{dv}{dt}ds \quad \left[\because a = \frac{dv}{dt}\right]$$

$$\text{or} \quad dW = m\frac{ds}{dt}dv = mvdv \quad \left[\because \frac{ds}{dt} = v\right]$$

Therefore, total work done on the body in order to increase its velocity from zero to v.

$$W = \int_0^v mvdv = m\int_0^v vdv = m\left[\frac{v^2}{2}\right]_0^v = \frac{1}{2}mv^2$$

This work done appears as kinetic energy of the body.

$$\text{i.e. Kinetic energy, KE} = \frac{1}{2}mv^2$$

Hence, the kinetic energy of a body is equal to one-half the product of the mass of the body and the square of its velocity.

Note

We observed that KE $\propto m$ and also KE $\propto v^2$. Thus, a heavier body and a body moving faster possess greater energy. The reverse is also true.

EXAMPLE |2| Rocket Propulsion

A toy rocket of mass 0.1 kg has a small fuel of mass 0.02 kg which it burns out in 3 s. Starting from rest on horizontal smooth track it gets a speed of 20 ms^{-1} after the fuel is burnt out. What is the approximate thrust of the rocket? What is the energy content per unit mass of the fuel? (Ignore the small mass variation of the rocket during fuel burning). **[NCERT]**

Sol. Here, $m = 0.1 \text{ kg}$, $u = 0$, $v = 20 \text{ ms}^{-1}$, $t = 3\text{s}$

$$\text{Thrust of the rocket} = ma = m\frac{v-u}{t}$$

$$= 0.1 \times \frac{20-0}{3} = \frac{2}{3}\text{N} \quad \left[\because v = u + at \text{ or } a = \frac{v-u}{t}\right]$$

Kinetic energy gained by the rocket

$$K = \frac{1}{2}mv^2$$

$$= \frac{1}{2} \times 0.1 \times (20)^2 = 20 \text{ J}$$

Energy content per unit mass of the fuel

$$= \frac{\text{Total energy}}{\text{Mass of the fuel}} = \frac{20 \text{ J}}{0.02 \text{ kg}} = 1000 \text{ Jkg}^{-1}$$

Relation between Kinetic Energy and Linear Momentum

According to linear momentum,

we know $\quad p = mv \quad$ ...(i)

where, m is mass and v is the velocity of a body

and kinetic energy of the body $= \frac{1}{2}mv^2 = \frac{1}{2m}(m^2v^2)$

$$\text{KE} = \frac{p^2}{2m} \qquad \text{[from Eq. (i)]}$$

or $$p^2 = 2m\,\text{KE}$$

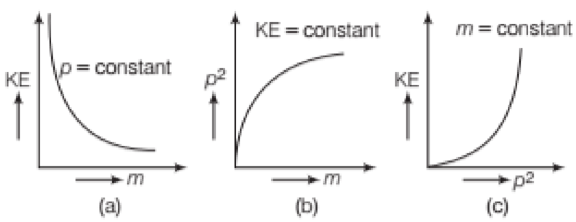
$\Rightarrow$ $\boxed{\text{Linear momentum, } p = \sqrt{2m\,\text{KE}}}$

Let us consider three cases as given below.

(i) Further, if $p =$ constant, $\text{KE} \propto \frac{1}{m}$. This is shown in Fig. (a).

(ii) If KE = constant, $p^2 \propto m$ or $p \propto \sqrt{m}$. This is shown in Fig. (b).

(iii) If $m =$ constant, $p^2 \propto \text{KE}$ or $p \propto \sqrt{\text{KE}}$. This is shown in Fig. (c).

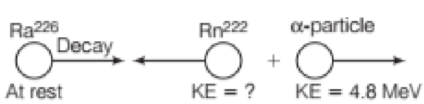


Graphical representation of kinetic energy and linear momentum

EXAMPLE |3| Radioactive Decay

A nucleus of radium ($^{226}_{88}$Ra) decays to $^{222}_{86}$Rn by the emission of α-particle ($^{4}_{2}$He) of energy 4.8 MeV. If mass of $^{222}_{86}$Rn = 222.0 amu and mass of $^{4}_{2}$He = 4.003 amu, then calculate the recoil energy of the daughter nucleus $^{222}_{86}$Rn. **[NCERT]**

Sol. The nuclear decay may be represented as follows



The kinetic energy of a particle is given by

$$K = \frac{p^2}{2m}$$

$\therefore$ $$p = \sqrt{2mK}$$

As momentum is conserved in the absence of an external force, so $\quad mK =$ constant

or $$m_{\text{Rn}}K_{\text{Rn}} = m_\alpha K_\alpha$$

or $$K_{\text{Rn}} = \frac{m_\alpha K_\alpha}{m_{\text{Rn}}} = \frac{4.003 \times 4.8}{222} = 0.0866 \text{ MeV}$$

WORK ENERGY THEOREM OR WORK ENERGY PRINCIPLE

It states that work done by the net force acting on a body is equal to the change produced in the kinetic energy of the body.

Work Energy Theorem for a Constant Force

When a force F acting on a body of mass m produces acceleration a in it. After covering distance s, suppose the velocity of the body changes from u to v. We use the equation of motion $v^2 - u^2 = 2as$

Multiply both sides by $\frac{1}{2}m$, we get

$$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = mas$$

By Newton's second law, $\quad F = ma$

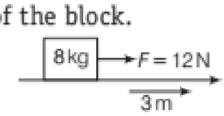
$\therefore$ $$\frac{1}{2}mv^2 - \frac{1}{2}mu^2 = Fs$$

or $$\boxed{K_f - K_i = W} \qquad [\because W = Fs]$$

where, K_f and K_i are the final and initial kinetic energies of the body.

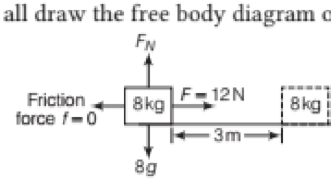
EXAMPLE |4| Pulling a Block

A 8 kg block initially at rest is pulled to the right along a frictionless horizontal surface by a constant, horizontal force of 12 N, as shown in figure. Find the velocity of the block after it has moved a distance of 3 m. Also, find out the acceleration of the block.



The weight of the block is balanced by the normal force and neither of these forces does work, since the displacement is horizontal. Because there is no friction, the resultant external force is 12 N.

Sol. First of all draw the free body diagram of the block.



Find out the work done by the force 12 N.

$$W = F\,s = Fs\cos 0° = 12 \times 3 = 36 \text{ J}$$

The work done by the normal and the weight of body will be zero.

If K_f and K_i are the final and initial kinetic energies of the block respectively, then according to work energy theorem.

$$W = K_f - K_i = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}mv^2$$

[$\because$ initial velocity of the block $u = 0$]

Velocity of the block,

$$v = \sqrt{\frac{2W}{m}} = \sqrt{\frac{2 \times 36}{8}} \quad [\because \text{given, } m = 8 \text{ kg}]$$

$$v = 3 \text{ m/s}$$

The acceleration of the block can be calculated using the kinematic equation.

$$v^2 = u^2 + 2as$$

$$\therefore \quad a = \frac{v^2 - u^2}{2s} = \frac{9}{2 \times 3} = 1.5 \text{ m/s}^2$$

A Block over Another Block

When a block A is placed on block B which is accelerating towards right with respect to ground as shown in figure. Then, work done by weight (w) and normal reaction (N) are zero because they act perpendicular to the displacement of block A.

In inertial frame, we have, $W_{\text{friction}} + W_{\text{pseudo}} = \Delta\text{KE}$,

if block A is at rest with respect to block B, then $\Delta\text{KE} = 0$ and therefore, $W_{\text{pseudo}} = -W_{\text{friction}}$.

In non-inertial frame,

$$W_{\text{external}} + W_{\text{internal}} + W_{\text{pseudo}} + W_{\text{other}} = \Delta\text{KE}$$

Work energy theorem is true for any system of particles in the presence of all types of force (conservative or non-conservative).

When KE increases, then work done is positive and when KE decreases, then work done is negative.

Work Energy Theorem for a Variable Force

Suppose a variable force **F** acts on a body of mass m and produces displacement ds in its own direction ($\theta = 0°$). Then,.the small work done is

$$dW = \mathbf{F} \cdot d\mathbf{s} = F ds \cos 0° = F ds$$

The time rate of change of kinetic energy is

$$\frac{dK}{dt} = \frac{d}{dt}\left[\frac{1}{2}mv^2\right]$$

$$= \frac{1}{2} \times 2v \times m\frac{dv}{dt}$$

$$= m\frac{dv}{dt}(v) = (ma)\,v$$

$$\left[\because \text{force, } F = ma \text{ and } v = \frac{ds}{dt}\right]$$

$$\therefore \quad \frac{dK}{dt} = F \cdot \frac{ds}{dt}$$

Thus, $\quad dK = F\,ds$

Integrating from the initial position (x_i) to final position (x_f), we have

$$\int_{K_i}^{K_f} dK = \int_{x_i}^{x_f} F\,ds$$

where, K_i and K_f are the initial and final kinetic energies corresponding to x_i and x_f, respectively.

or $\quad$ Change in kinetic energy, $K_f - K_i = \int_{x_i}^{x_f} F\,ds$

$$\therefore \quad K_f - K_i = \text{work done on the body } (W)$$

$$= \text{increase in KE of body.}$$

This proves the work energy theorem for a variable force.

Note

- The work energy theorem is not independent of Newton's second law.
- Newton's second law in two or three dimensions is in vector form, but the work energy theorem is in scalar form.

EXAMPLE |5| When a Pebble Hits the Ground

Consider a drop of small pebble of mass 1.00 g falling from a cliff of height 1.00 km. It hits the ground with a speed of 50.0 ms^{-1}. What is the work done by the unknown resistive force? **[NCERT]**

Sol. We assume that the pebble is initially at rest on the cliff.

$$\therefore \quad u = 0,\ m = 1.00 \text{ g} = 10^{-3} \text{ kg}$$

$$v = 50 \text{ ms}^{-1},\ h = 1.00 \text{ km} = 10^3 \text{ m}$$

The change in KE of the pebble is

$$\Delta K = \frac{1}{2}mv^2 - \frac{1}{2}mu^2$$

$$= \frac{1}{2} \times 10^{-3} \times (50)^2 - 0 = 1.25 \text{ J}$$

Assuming that $g = 10 \text{ ms}^{-2}$ is constant, the work done by the gravitational force is

$$W_g = F \cdot h = mgh = 10^{-3} \times 10 \times 10^3 = 10.0 \text{ J}$$

If W_r is the work done by the resistive force on the pebble, then from the work energy theorem,

$$\Delta K = W_g + W_r$$

or $\quad W_r = \Delta K - W_g$

$$= 1.25 - 10.0 = -8.75 \text{ J}$$

EXAMPLE |6| A Stretched Block on a Horizontal Plane

A block of mass $m = 1$ kg, moving on a horizontal surface with speed $v_i = 2\ \text{ms}^{-1}$ enters a rough patch ranging from $x = 0.10$ m to $x = 2.01$ m. The retarding force F_r on the block in this range is inversely proportional to x over this range.

$$F_r = \frac{-k}{x}\ 0.1 < x < 2.01 \text{ m}$$
$$= 0 \text{ for } x < 0.1 \text{ m and } x > 2.01 \text{ m}$$

where, $k = 0.5$ J. What is the final kinetic energy and speed v_f of the block as it crosses this patch? **[NCERT]**

Sol. By work energy theorem,

$$\Delta K = W_r \quad \text{or} \quad K_f - K_i = \int_{x_i}^{x_f} F_r\, dx$$

$$\therefore \quad K_f = K_i + \int_{0.1}^{2.01} \frac{(-k)}{x}\, dx \qquad [\because F_r = -kx]$$

where, k = spring constant

$$K_f = \frac{1}{2} m v_i^2 - k \int_{0.1}^{2.01} \frac{1}{x} dx$$
$$= \frac{1}{2} \times 1 \times 2^2 - k\, [\ln x]_{0.1}^{2.01}$$
$$= 2 - 0.5 \left[\ln \frac{2.01}{0.1}\right] = 2 - 0.5 \ln 20.1$$
$$= 2 - 0.5 \times 2.303 \log 20.1$$
$$= 2 - 0.5 \times 2.303 \times 1.3032$$
$$K_f = 2 - 1.5 = 0.5 \text{ J}$$

$$\text{Final speed, } v_f = \sqrt{\frac{2K_f}{m}} = \sqrt{\frac{2 \times 0.5}{1}} = 1\ \text{ms}^{-1}$$

POTENTIAL ENERGY

The potential energy of a body is defined as the energy possessed by the body by virtue of its position or configuration in some field.

Thus, the potential energy of a system that can be associated with the configuration of a system of objects that exert forces on one another. So, configuration of the system changes, then its potential energy changes.

e.g.

(i) A body lying on the roof of a building has some potential energy. When allowed to fall down it can do work.

(ii) When a spring is compressed or stretched, work done in compressing or stretching spring is stored in the spring in the form of potential energy.

Units and Dimensions of Potential Energy

The units and dimensions of potential energy are same as that of kinetic energy given as,

Dimensions = $[\text{ML}^2\text{T}^{-2}]$

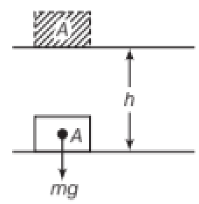
Unit = Joule (J) in SI

> ***Potential Energy Converted into Kinetic Energy***
>
> The notion of potential energy applies to only those forces where the work done against the force gets stored up as energy by virtue of position or configuration of the body when external constraints are removed.
>
> This energy appears as kinetic energy, when the position or configuration of the body gets change under the action of external constraints.

Gravitational Potential Energy

Gravitational potential energy of a body is the energy possessed by the body by virtue of its position above the surface of the earth.

Consider a body of mass m lying on the surface of the earth as shown in figure. Let g be the acceleration due to gravity at this place. For height much smaller than the radius of the earth ($h << R_E$) the value of g can be taken constant.



Gravitational potential energy of a body

Force needed to lift the body up with zero acceleration,

$$F = \text{weight of the body} = mg$$

Work done on the body in raising it through height h,

$$W = Fh = mgh$$

This work done against gravity is stored as the gravitational potential energy (U) of the body.

$$\therefore \quad \boxed{\text{Gravitational potential energy, } U = mgh}$$

If h is taken as a variable, then the gravitational force F equal to the negative of derivative of U with respect to h.

$$\text{Thus,} \qquad F = -\frac{d(U)}{dh} = -mg$$

Here, the negative sign indicates that the gravitational force is downward.

Moreover, if a body of mass m is released from rest, from the top of a smooth (frictionless) inclined plane of height h, just before it hits the ground, its speed is given by the kinematic relation.

$$v^2 - (0)^2 = 2gh \text{ or } v = \sqrt{2gh}$$

So, KE acquired by the body on reaching the ground,

$$\frac{1}{2} m v^2 = \frac{1}{2} m \times 2gh = mgh$$

= work done by the gravitational force

This equation shows that the gravitational potential energy of the object at height h, when it is released reveals itself as kinetic energy of the object on reaching the ground

$$v = \sqrt{2gh}$$

- *For gravitational potential energy, the zero of potential energy is chosen to be the ground.*
- *The work done by the gravitational force does not depend on the angle of inclination of the inclined plane or the path of the falling body. It just depends on the initial and final position. Thus, gravitational force is a conservative force.*

$$\therefore \qquad F(x) = \frac{-dU}{dx}$$

If x_i and x_f are the initial and final position of the object and U_i and U_f are the corresponding potential energies, then

$$\int_{x_i}^{x_f} F(x)\, dx = -\int_{U_i}^{U_f} dU = U_i - U_f$$

EXAMPLE |7| A Ball Hits the Ground

A ball falls from a height of 20 m. Find out of the velocity with which the ball hits the ground?

Sol. Given, $h = 20\text{m}$, $v = ?$

When the ball hits the ground its kinetic energy is converted into potential energy.

$\therefore$ When it hits the ground KE = PE

$$\frac{1}{2}mv^2 = mgh$$

$$v = \sqrt{2gh} = \sqrt{2 \times 9.8 \times 20} = \sqrt{392} = 19.798 \text{ m/s}$$

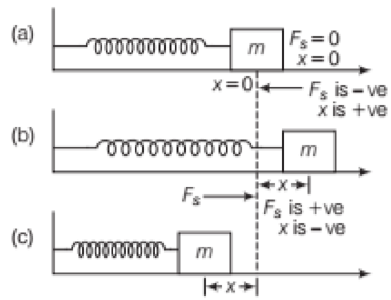
Potential Energy of a Spring

There are many types of spring. Important among these are helical and spiral springs as shown in figure.

(a) Helical spring and (b) Spiral spring

Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of elastic potential energy of the spring. Thus, potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

Consider an elastic spring of negligibly small mass with its one end attached to a rigid support and its other end is attached to a block of mass m can slide over a smooth horizontal surface. At $x = 0$, the position is in the state of equilibrium as shown in Fig. (a), when the spring is stretched in Fig. (b) and compressed by pushing the block as shown in Fig. (c). So, spring force F_s begins to act in equilibrium position.



Equilibrium, stretched and compressed states of an elastic spring

For a small stretch or compression, spring obeys Hooke's law, i.e. restoring force ∝ stretch or compression

$$-F_s \propto x \quad \Rightarrow \quad F_s = -kx$$

where, k is called **spring constant**. Its SI unit is N/m.The negative sign shows F_s acts in the opposite direction of displacement x. The work done by the spring force for the small extension is $dW_s = F_s\, dx = -kx\, dx$

If the block is moved from an initial displacement x_i to final displacement x_f, then work done by spring force is

$$W_s = \int dW_s = -\int_{x_i}^{x_f} kx\, dx = -k\left[\frac{x^2}{2}\right]_{x_i}^{x_f}$$

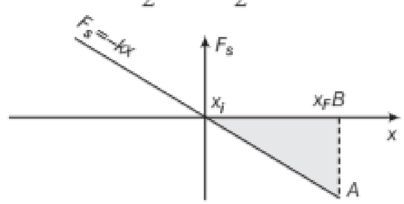
or $\boxed{\text{Net work done by the spring, } W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_f^2}$

(*i*) If $x_i = 0$ (i.e. for mean position) then, work done by the spring force is

$$W_s = -\frac{1}{2}kx_f^2$$

(*ii*) If the block is pulled from x_i and allowed to return to x_i, then

$$W_s = \frac{1}{2}kx_i^2 - \frac{1}{2}kx_i^2 = \text{zero}$$



Plot of F_s *versus* x

Conclusion

(i) Spring force F_s is position dependent as it is clear in Hooke's law.
i.e. $F_s = -kx$

(ii) The work done by the spring force depends on initial and final position.

(iii) The work done by the spring force in a cyclic process is zero.

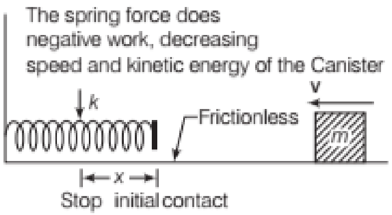
(iv) It can be expressed as the difference between the initial and final values of a potential energy function. Hence, spring force is a **conservative force**.

Note

The potential energy of a body which is subjected to a conservative force is uncertain upto a certain limit. This is because the point of zero potential energy is a matter of choice. For the spring potential energy $\frac{1}{2}kx^2$, the zero of the potential energy is the equilibrium position of the oscillating mass.

EXAMPLE |8| Compresses a Spring

As shown in figure, a canister of mass $m = 0.40$ kg slides across a horizontal frictionless counter with speed $v = 0.50$ m/s. It then runs into and compresses a spring of spring constant $k = 750$ N/m. When the canister is momentarily stopped by the spring, by what distance d is the spring compressed ?



The work W_s done on the canister by the spring force is related to the distance x by the relation $W_s = -\frac{1}{2}kx^2$.

The work W_s is also related to the kinetic energy of the canister by $(K_f - K_i = W)$

The canister's kinetic energy has an initial value of $K = \frac{1}{2}mv^2$ and a value of zero when the canister is momentarily at rest.

Sol. Write the work energy theorem for the canister as

$$K_f - K_i = \frac{1}{2}kx^2$$

Substituting according to the third key idea gives us the expression

$$0 - \frac{1}{2}mv^2 = -\frac{1}{2}kx^2$$

$$x = v\sqrt{\frac{m}{k}} = (0.50 \text{ m/s})\sqrt{\frac{0.40 \text{ kg}}{750 \text{ N/m}}}$$

$$= 0.50 \times 0.02 = 0.0115 \text{ m} = 1.15 \text{ cm}$$

MOTION IN A VERTICAL CIRCLE

Suppose a particle of mass m is attached to an inextensible light string of length L. The particle is moving in a vertical circle of radius L about a fixed point O.

It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point B, the circle as shown in figure. Here we have $h = R(1 - \cos\theta)$

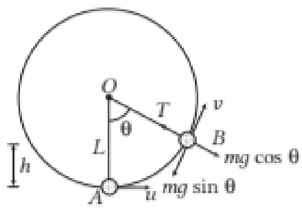
Now from conservation of mechanical energy, we have

$$\frac{1}{2}m(u^2 - v^2) = mgh \qquad \ldots\text{(i)}$$

The necessary centripetal force is provided by the resultant of tension T and $mg\cos\theta$.

$$\therefore \qquad T - mg\cos\theta = \frac{mv^2}{R} \qquad \ldots\text{(ii)}$$

As speed of the particle decreases with height, tension in the string is maximum at the bottom. The particle will complete the circle if the string does not slack even at the highest point.



Motion of a particle in vertical circle

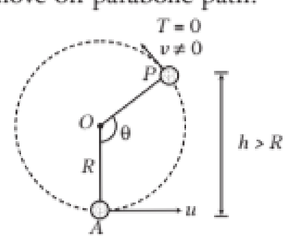
Now following conclusions can be made using above Eqs. (i) and (ii).

(i) **Minimum velocity at highest point** so that particle complete the circle $v_{\min} = \sqrt{gL}$, at this velocity, tension in the string is zero.

(ii) **Minimum velocity at lowest point** so that particle complete the circle, $v_{\min} = \sqrt{5gL}$, at this velocity, tension in the string is 6 mg.

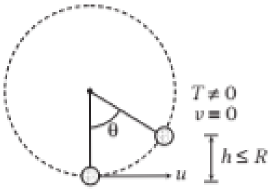
(iii) **When string is horizontal**, then minimum velocity is $\sqrt{3Rg}$ and tension in this condition is $3mg$.

(iv) If velocity at lowest point is less than $\sqrt{5gR}$, then tension in the string becomes zero before reaching the highest point, now the particle will leave the circle and will move on parabolic path.



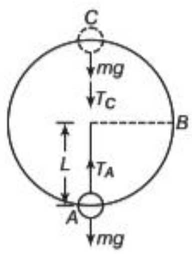
In this condition, if $\sqrt{2gR} < v < \sqrt{5gR}$ then tension in the string becomes zero but velocity is not zero, the particle will leave circle at $90° < \theta < 180°$ or $h > R$

(v) If velocity at lowest point is $0 < v \leq \sqrt{2gR}$, the particle will oscillate, in this condition velocity becomes zero but tension is not zero. The particle will oscillate in lower half of circle, *i.e.*, $0° < \theta < 90°$



EXAMPLE |9| A Ballistic Bob

A bob of mass m is suspended by a light string of length L. It is imparted a horizontal velocity v_0 at the lowest point A such that it completes a semi-circular trajectory in the vertical plane with the string becoming slack only on reaching the topmost point, C. This is shown in figure obtain an expression for (i) v_0 (ii) the speeds at points B and C, (iii) the ratio of the kinetic energies (K_B/K_C) at B and C. Comment on the nature of the trajectory of the bob after it reaches the point C. [NCERT]



Sol. (i) Two external forces act on the bob, i.e. gravity and tension (T) in the string.

At the lowest point A, the potential energy of the system can be taken zero. So, at point A,

Total mechanical energy = Kinetic energy

$$E = \frac{1}{2} mv_0^2 \quad \text{...(i)}$$

If T_A is the tension in the string at point A, then from Newton's second law,

$$T_A - mg = \frac{mv_0^2}{L} \quad \text{...(ii)}$$

At the highest point C, the string slackens, so the tension T_C becomes zero. If v_C is the speed at point C, then by conservation of energy,

$$E = K + U \text{ or } E = \frac{1}{2} mv_C^2 + 2\,mgL \quad \text{...(iii)}$$

From Newton's second law,

$$mg = \frac{mv_C^2}{L} \quad \text{...(iv)}$$

or

$$mv_C^2 = mgL \quad \text{...(v)}$$

Using Eq. (v) in Eq. (iii), we get

$$E = \frac{1}{2} mgL + 2\,mgL = \frac{5}{2} mgL \quad \text{...(vi)}$$

From Eqs. (i) and (vi), we get

$$\frac{5}{2} mgL = \frac{m}{2} v_0^2 \text{ or } v_0 = \sqrt{5\,gL} \quad \text{...(vii)}$$

(ii) From Eq. (iv), we have $v_C = \sqrt{gL}$

The total energy at B is $E = \frac{1}{2} mv_B^2 + mgL$ ...(viii)

From Eqs. (i) and (viii), we get

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} mv_0^2$$

$$\frac{1}{2} mv_B^2 + mgL = \frac{1}{2} m \times 5gL \quad \text{[using Eq. (vii)]}$$

$$\therefore \quad v_B = \sqrt{3gL}$$

(iii) The ratio of the kinetic energies at B and C is

$$\frac{K_B}{K_C} = \frac{(1/2)\,mv_B^2}{(1/2)\,mv_C^2} = \frac{3}{1} = 3:1$$

VARIOUS FORMS OF ENERGY

Energy can manifest itself in many forms. Some of these forms are as follows

(i) **Mechanical Energy** The sum of kinetic and potential energies is called as **mechanical energy.** KE is due to motion while the potential energy is due to position or configuration.

(ii) **Internal Energy** When the molecules of a body vibrate with respect to one another, the molecules possess potential energy due to their location against the intermolecular forces and possess kinetic energy because of motion. The sum of these kinetic and potential energies of all the molecules constituting the body is called its **internal energy.**

(iii) **Heat Energy** A body possess heat energy due to the disorderly motion of its molecules. Heat energy is also related to the internal energy of the body. In winter, we generate heat by rubbing our hands against each other.

(iv) **Chemical Energy** A stable chemical compound has lesser energy than its constituent atoms, the difference being in the arrangement and motion of electron in the compound. This difference is called **chemical energy.**

If the total energy of the reactant is more than the product of the reaction, then heat is released and the reaction is said to be an **exothermic reaction.** If the reverse is true, then heat is absorbed and the reaction is **endothermic.**

(v) **Electrical Energy** Work is said to be done when electric charge moves from one point to another in an electric field or motion of a current carrying conductor inside a magnetic field. The energy is associated with an electric current. The flow of current causes bulbs to glows motor to run and electric heater to produce heat.

(vi) **Nuclear Energy** When U^{235} nucleus breaks up into lighter nuclei on being bombarded by a slow neutron, a tremendous amount of energy is released. Thus, the energy so released is called nuclear energy and this phenomenon is known as **nuclear fission. Nuclear reactors** and **nuclear bombs** are the sources of nuclear energy.

Equivalence of Mass and Energy

In 1905, Einstein discovered that mass can be converted into energy and *vice-versa*. He showed that mass(m) and energy(E) are equivalent and related by the relation given,

$$\boxed{E = mc^2}$$

where, m is mass that disappears. E is energy that appears and c is velocity of light in vacuum.

Conversely, when energy E disappears, a mass $m \, (= E/c^2)$ appears. Thus, according to modern physics, mass and energy are not conserved separately, but are conserved as a single entity called **mass energy**. So, the law of conservation of mass and law of conservation of energy have been unified by this relation into a single law of conservation of mass energy.

As speed of light in vacuum is approximately 3×10^8 m/s. Thus, a staggering amount of energy is associated with a mere one kilogram of matter.

i.e. $$E = 1 \times (3 \times 10^8)^2 = 9 \times 10^6 \text{ J}$$

Thus, it is equivalent to the annual electrical output of a large (3000 MW) power generating station.

EXAMPLE |10| Nuclear Fusion Reaction

Estimate the amount of energy released in the nuclear fusion reaction

$$_1H^2 + {_1H^2} \longrightarrow {_2He^3} + {_0n^1}$$

Given that $M\,(_1H^2) = 2.0141$ u,

$M\,(_2He^3) = 3.0160$ u

Mn $= 1.0087$ u, where 1 u $= 1.661 \times 10^{-27}$ kg

Express your answer in units of MeV.

Sol. Given, $_1H^2 = 2.014$ u, $_2He^3 = 3.0160$ u, Mn $= 1.0087$ u

$1 \text{ u} = 1.66 \times 10^{-27}$ kg

Energy released = ?

mass of reactant, mass$_r$ $= 2 \times 2.0141 = 4.0282$ u

mass of products, mass$_p$ $= 3.0160 + 1.0087 = 4.0247$ u

Loss of mass = mass$_p$ − mass$_r$

$$\Delta M = (4.0282 - 4.0247) \text{ u}$$
$$= 0.0035 \text{ u}$$
$$= 0.0035 \times 1.66 \times 10^{-27} \text{ kg}$$

Energy released $E = (\Delta m)c^2$

$$E = 0.0035 \times 1.66 \times 10^{-27} \times (3 \times 10^8)^2$$
$$= 52.2 \times 10^{-14} \text{ J}$$

As we know 1 MeV $= 1.602 \times 10^{-13}$ J

$$\text{Energy released} = \frac{52.2 \times 10^{-14}}{1.602 \times 10^{-13}} = 3.26 \text{ MeV}$$

Table below gives an approximate energy associated with some of the important phenomena.

Energy Associated with Some Important Phenomena

Description	Energy (J)
Big Bang	10^{68}
Radio energy emitted by the galaxy during its lifetime	10^{55}
Rotational energy of the Milky Way	10^{52}
Energy released in a supernova explosion	10^{44}
Ocean's hydrogen in fusion	10^{34}
Rotational energy of the earth	10^{29}
Annual solar energy incident on the earth	5×10^{24}
Annual wind energy dissipated near earth's surface	10^{22}
Annual global energy usage by human	3×10^{20}
Annual energy dissipated by the tides	10^{20}
Energy release of 15-megaton fusion bomb	10^{17}
Annual electrical output of large generating plant	10^{16}
Thunderstorm	10^{15}
Energy released in burning 1000 kg of coal	3×10^{10}
Kinetic energy of a large jet aircraft	10^9
Energy released in burning 1 L of gasoline	3×10^7
Daily food in take of a human adult	10^7
Work done by a human heart per beat	0.5
Turning this page	10^{-3}
Flea hop	10^{-7}
Discharge of a single neutron	10^{-10}
Typical energy of a proton in a nucleus	10^{-13}
Typical energy of an electron in an atom	10^{-18}
Energy to break one bond in DNA	10^{-20}

Principle of Conservation of Energy

It states that, the energy can neither be created nor be destroyed but can only be converted from one form to another.

It is one of the fundamental laws and is applied in all the processes taking place in the universe.

Whenever energy in one form disappears, then equivalent amount of energy appears in some other form. Thus, the total energy remains constant. Therefore, principle of conservation of energy may be stated as below

Total energy of an isolated system always remains constant. Since, the universe as a whole may be viewed as an isolated system, total energy of the universe is constant. If one part of the universe loses energy, then other part must gain an equal amount of energy.

The principle of conservation of energy cannot be proved as such. However, no violation of this principle has been observed.

EXAMPLE |11| Life Span of a Nuclei

The nucleus Fe^{57} emits a γ-ray of energy 14.4 keV. If the mass of the nucleus is 56.935 amu, calculate the recoil energy of the nucleus. [Take, 1 amu $= 1.66 \times 10^{-27}$ kg]

[NCERT]

Sol. The nuclear decay may be represented as follows

$$\underset{\text{Excited state}}{Fe^{57}} \longrightarrow \underset{\text{Ground state}}{Fe^{57}} + h\nu \ (\gamma\text{-ray photon})$$

According to de-Broglie hypothesis, momentum of a photon of energy E is

$$p = \frac{E}{c} = \frac{14.4 \text{ keV}}{c}$$

$$= \frac{14.4 \times 1.6 \times 10^{-16} \text{ J}}{3 \times 10^8 \text{ ms}^{-1}}$$

$$p = 7.68 \times 10^{-24} \text{ kg ms}^{-1}$$

By conservation of momentum, the momentum of daughter nucleus, p = momentum of γ-ray photon

$$= 7.68 \times 10^{-24} \text{ kg ms}^{-1}$$

The recoil energy of the nucleus will be

$$K = \frac{p^2}{2m} = \frac{(7.68 \times 10^{-24})^2}{2 \times 56.935 \times 1.66 \times 10^{-27}}$$

$$= 0.32 \times 10^{-21} \text{ J} = \frac{0.312 \times 10^{-21}}{1.6 \times 10^{-16}} \text{ keV}$$

$$K = 1.95 \times 10^{-6} \text{ keV}$$

TOPIC PRACTICE 2

OBJECTIVE Type Questions

1. An object of mass 10 kg is moving with velocity of 10 ms^{-1}. A force of 50 N acted upon it for 2 s. Percentage increase in its KE is

(a) 25% (b) 50%
(c) 75% (d) 300%

Sol. (d) Initial velocity $= 10 \text{ ms}^{-1}$

$$\text{Final velocity} = \frac{50}{10} \times 2 + 10 = 20 \text{ ms}^{-1}$$

$$\left(\text{Acceleration} = \frac{50}{10} = 5 \text{ m/s}^2\right)$$

$$\text{Initial KE} = \frac{1}{2} \times 10 \times 10 \times 10 = 5 \times 10^2 \text{ J}$$

$$\text{Final KE} = \frac{1}{2} \times 10 \times 20 \times 20 = 20 \times 10^2 \text{ J}$$

$$\% \text{ increase} = \frac{(20-5) \times 10^2}{5 \times 10^2} \times 100 = 300\%$$

2. Two masses of 1 g and 4 g are moving with equal kinetic energy. The ratio of the magnitudes of their momentum is

(a) 4 : 1 (b) $\sqrt{2}:1$
(c) 1 : 2 (d) 1 : 16

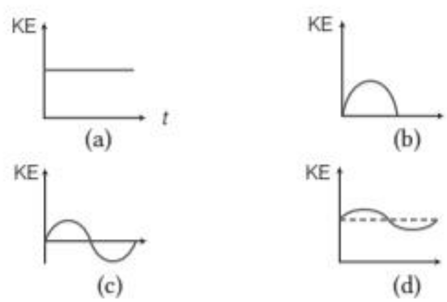
Sol. (c) As we know that linear momentum $= \sqrt{2mK}$

$$\left(\because K = \frac{p^2}{2m}\right)$$

For same kinetic energy, $p \propto \sqrt{m}$

$$\frac{p_1}{p_2} = \sqrt{\frac{m_1}{m_2}} = \sqrt{\frac{1}{4}} = \frac{1}{2} = 1:2$$

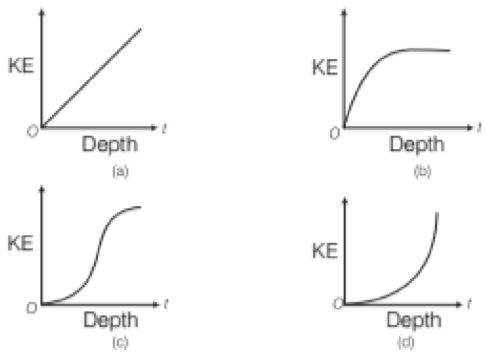
3. Which of the diagrams shown in figure most closely shows the variation in kinetic energy of the earth as it moves once around the sun in its elliptical orbit? **[NCERT Exemplar]**



Sol. (d) When the earth is closest to the sun, speed of the earth is maximum, hence, KE is maximum. When the earth is farthest from the sun speed is minimum hence, KE is minimum but never zero and negative.

This variation is correctly represented by option (d).

4. Which of the diagrams in figure correctly shows the change in kinetic energy of an iron sphere falling freely in a lake having sufficient depth to impart it a terminal velocity? **[NCERT Exemplar]**



Sol. (b) First velocity of the iron sphere increases and after sometime becomes constant, called terminal velocity. Hence, accordingly first KE increases and then becomes constant which is best represented by (b).

VERY SHORT ANSWER Type Questions

5. Is it possible to exert a force which does work on a body without changing its kinetic energy. If, so give example.

Sol. Yes, when spring is compressed or when a body is pulled with a constant velocity on a rough horizontal surface.

6. What is the source of kinetic energy of the bullet coming out of the bullet of a rifle?

Sol. The source of kinetic energy of bullet is the potential energy of the compressed spring in the loaded rifle.

7. When an air bubble rises in water, what happens to its potential energy?

Sol. Potential energy of air bubble decreases because work is done by upthrust on the bubble.

8. A spring is cut into two equal halves. How is the spring constant of each half affected?

Sol. Spring constant of each half becomes twice the spring constant of the original spring.

9. What type of energy stored in (i) the wound spring of a watch (ii) a stretched bow while ready to project an arrow?

Sol. In both cases, energy is stored in the form of elastic potential energy due to change in configuration.

10. Does potential energy of a spring decrease/increase when it is compressed or stretched?

Sol. When a spring is compressed or stretched, potential energy of the spring increases in both the cases. This is because work is done by us in compression as well as stretching.

SHORT ANSWER Type Questions

11. A bolt of mass 0.3 kg falls from the ceiling of an elevator moving down with an uniform speed of $7\ \text{ms}^{-1}$. It hits the floor of the elevator (length of the elevator = 3 m) and does not rebound. What is the heat produced by the impact? Would your answer be different if the elevator were stationary? **[NCERT]**

Sol. Given, mass of bolt, $m = 0.3$ kg, height through which bolt falls $h = 3$ m.

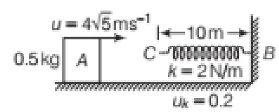
When the bolt falls on floor of elevator and does not rebound, it suffers a loss of potential energy

$$\Delta U = mgh = 0.3 \times 9.8 \times 3 = 8.82\ \text{J}$$

$\therefore$ Amount of heat produced = Loss of potential energy = 8.82 J, because there is no gain in KE at all.

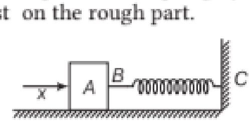
The answer is true when either the elevator is at rest or in state of uniform motion because potential energy of bolt has no connection with velocity of elevator.

12. A 0.5 kg block slides from the point A on a horizontal track with an initial speed $4\sqrt{5}\ \text{ms}^{-1}$ towards a weightless horizontal spring of length 10 m and spring constant 2 N/m.



The initial track is frictionless and part BC under the unstretched length of spring has coefficient of kinetic friction $\mu_k = 0.2$, Calculate total distance by which the block move before coming finally to rest. ($g = 10\ \text{ms}^{-2}$).

Sol. Let the block compresses the spring by x before finally coming to rest on the rough part.



Total energy of the block on friction surface

= Work done against friction + Potential energy stored in spring.

$$\frac{1}{2} \times 0.5 \times 16 \times 5 = \mu_k\ mg\ s + \frac{1}{2} kx^2$$

$$20 = 0.2 \times 0.5 \times 10 \times x + \frac{1}{2} \times 2 \times x^2$$
$$20 = x + x^2$$
$$x^2 + x - 20 = 0$$
$$x^2 + 5x - 4x - 200 = 0$$
$$x(x+5) - 4(x+5) = 0$$
$$x = 4,\ x = -5$$

Hence, the compression of spring is 4 m.

13. Can a body have energy without momentum? If yes, then explain how they are related with each other?

Sol. Yes, when $p = 0$,

Then, $K = \frac{p^2}{2m} = 0$

But $E = K + U = U$(potential energy), which may or may not be zero.

14. A spring balance reads forces in Newtons. The scale is 20 cm long and read from 0 to 60 N. Find potential energy of spring when the scale reads 20 N.

Sol. We can calculate the spring constant of spring, as it is extended by 20 cm under 60 N force.

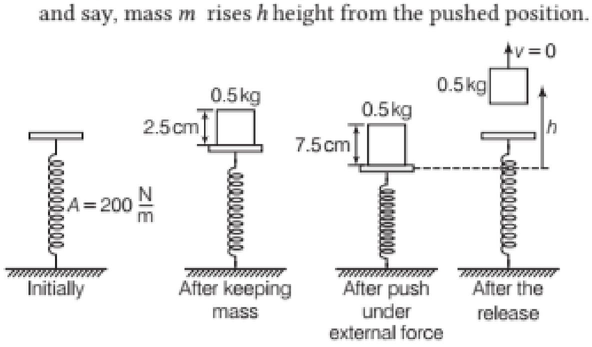
$$F = kx \Rightarrow 60 = k \times 20 \times 10^{-2}$$
$$k = 300\ \text{N/m}$$

At a force of 20 N, the extension in spring is

$$F = kx \Rightarrow 20 = 300x$$
$$x = \frac{2}{30} = \frac{1}{15}\ \text{m}$$

15. A vertical spring with constant 200 N/m has a light platform on its top. When a 500 g mass is kept on the platform spring compresses 2.5 cm. Mass is now pushed down 7.50 cm further and released. How far above later position will the mass fly? ($g = 10\ \text{ms}^{-2}$).

Sol. When the external force is removed after the push, the mass gets detached when spring obtain its natural length and say, mass m rises h height from the pushed position.



Loss in potential energy of spring = Gain in gravitational potential energy

$$\frac{1}{2}k[x^2] = mgh$$
$$\frac{1}{2} \times 200\,[0.1]^2 = 0.5 \times 10 \times h$$
$$1 = 5h \Rightarrow h = 0.2\ \text{m}$$

16. A body of mass 1.0 kg initially at rest is moved by a horizontal force of 0.5 N on a smooth frictionless table. Calculate the work done by the force in 10 s and show that this is equal to the change in kinetic energy of the body.

Sol. Acceleration of a body,

$$a = F/m = \frac{0.5}{1.0} = 0.5\ \text{ms}^{-2}$$

Distance travelled, $s = ut + \frac{1}{2}at^2$

$$s = 0 + \frac{1}{2} \times 0.5\,(10)^2 = 25\ \text{m}$$

Work done $= F \times s = 0.5 \times 25 = 12.5$ J

$$v = u + at = 0 + 0.5 \times 10 = 5\ \text{ms}^{-1}$$

Change in KE $= (1/2)\,m\,(v^2 - u^2)$

$$= (1/2) \times 1.0\,(5^2 - 0) = 12.51$$

17. A body of mass 0.5 kg travels in a straight line with velocity $v = ax^{3/2}$, where $a = 5\ \text{m}^{-\frac{1}{2}}\text{s}^{-1}$. What is the work done by the net force during its displacement from $x = 0$ to $x = 2$ m? **[NCERT]**

Sol. Given, $m = 0.5$ kg, $v = ax^{\frac{3}{2}}$, where $a = 5\ \text{m}^{-\frac{1}{2}}\ \text{s}^{-1}$.

We know that work done by net force acting on a body is equal to its change in KE. If velocity of body corresponding to $x = 0$ and $x = 2$ m be u and v respectively, then

$$W = \frac{1}{2}mv^2 - \frac{1}{2}mu^2 = \frac{1}{2}m(v^2 - u^2)$$

Now, $u = a \cdot (0)^{3/2} = 0$ and $v = a \cdot (2)^{3/2}$

$$\therefore \quad W = \frac{1}{2} \times 0.5\,[\{a(2)^{3/2}\}^2 - 0]$$
$$= \frac{1}{2} \times 0.5 \times a^2 \times (2)^3$$
$$= \frac{1}{2} \times 0.5 \times (5)^2 \times 8 = 50\ \text{J}$$

LONG ANSWER Type I Questions

18. The bob of a pendulum is released from a horizontal position. If the length of the pendulum is 1.5 m, what is the speed with which the bob arrives at the lowermost point, given that it dissipated 5% of its initial energy against air resistance? **[NCERT]**

Sol. On releasing the bob of pendulum from horizontal position, it falls vertically downward by a distance equal to length of pendulum i.e. $h = l = 1.5$ m.

As 5% of loss in PE is dissipated against air resistance, the balance 95% energy is transformed into KE. Hence,

$$\frac{1}{2}mv^2 = \frac{95}{100} \times mgh$$

$$\Rightarrow \quad v = \sqrt{2 \times \frac{95}{100} \times gh}$$

$$= \sqrt{\frac{2 \times 95 \times 9.8 \times 1.5}{100}}$$

$$= 5.3 \text{ ms}^{-1}$$

19. Underline the correct alternative.

(i) When a conservative force does positive work on a body, the potential energy of the body increases decreases/remains unaltered.

(ii) Work done by a body against friction always results in a loss of its kinetic/potential energy.

(iii) The rate of change of total momentum of a many particle system is proportional to the external force/ sum of the internal forces on the system. [NCERT]

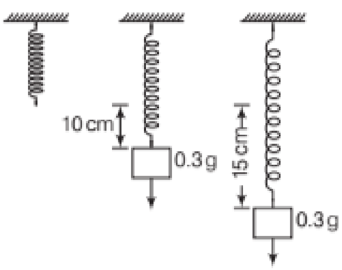
Sol. (i) The potential energy decreases, because $\Delta U = -\int F \cdot dx$. If work done by conservative force is positive, then obviously ΔU is – ve.

(ii) Friction always opposes motion. Hence, work done by a body against friction causes loss in its kinetic energy.

(iii) The total (net) momentum can change only when some net external force acts on the system. Hence, its rate of change is proportional to the external force.

20. When a 300 g mass is hung from a vertical spring, it stretches from equilibrium by 10 cm. What work is required to stretch it by next 5 cm?

Sol. We can calculate spring constant of spring by first information.



Work done by a vertical spring is

$$0.3\text{ g} = k[10 \times 10^{-2}]$$

$$2 = k \times 0.1 \Rightarrow k = \frac{3}{0.1} = 30 \frac{\text{N}}{\text{m}}$$

Extra work required to stretch it by next 5 cm.

$$W = \frac{1}{2}kx_2^2 - \frac{1}{2}kx_1^2$$

$$W = \frac{1}{2} \times 30\,[(15 \times 10^{-2})^2 - (10 \times 10^{-2})^2]$$

$$W = 15\,[225 - 100] \times 10^{-4} = 15 \times 125 \times 10^{-4} \text{ J}$$

$$W = 0.1875 \text{ J}$$

21. If momentum of a body increased by 300%, then what will be percentage increase in momentum of a body?

Sol. Consider a particle of mass m moving with a velocity v so, that its KE $= \frac{1}{2}mv^2$ and momentum, $p = mv$.

Thus, $\quad \text{KE} = \frac{p^2}{2m}$ or $p = \sqrt{2m\text{KE}}$

when KE is increased by 300%, then new KE

$\text{KE}' = \text{KE} + 300\%$ of $E = \text{KE} + 3\text{KE} = 4\text{KE}$

New momentum, $p' = \sqrt{2m\text{KE}'} = \sqrt{2m \times 4\text{KE}}$

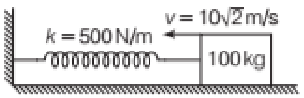
$$= 2\sqrt{2m\,\text{KE}} = 2p$$

∴ Percentage increase in momentum

$$= \frac{p' - p}{p} \times 100 = \frac{2p - p}{p} \times 100 = 100\%$$

22. A long spring of spring constant 500 N/m is attached to a wall horizontally and surface below the spring is rough with coefficient of friction 0.75. A 100 kg mass block moving with a speed $10\sqrt{2}$ ms^{-1} strikes the spring. Find the maximum compression of the spring. $(g = 10 \text{ ms}^{-2})$

Sol. When the block strikes the spring it carry some kinetic energy and all of that is spent against friction and stores in the compressed spring as potential energy.



Let the spring compresses by x.

Loss in kinetic energy of block = Gain in potential energy of the spring + Work done against friction.

$$\frac{1}{2}mv^2 = \frac{1}{2}kx^2 + \mu mg\,x$$

$$\frac{1}{2} \times 100 \times 200 = \frac{1}{2}500\,x^2 + 0.75 \times 100 \times 10 \times x$$

$$50 \times 200 = 250x^2 + 750x \Rightarrow 200 = 5x^2 + 15x$$

$$5x^2 + 15x - 200 = 0$$

$$x^2 + 3x - 40 = 0$$

$$x^2 + 8x - 5x - 40 = 0 \Rightarrow x(x + 8) - 5(x + 8) = 0$$

$$x = 5 \text{ m}, x = -8 \text{ m}$$

So, $\quad x = 5$ m

23. An electron and a proton are detected in a cosmic ray experiment, the first with kinetic energy 10 keV and the second with 100 keV. Which is faster, the electron or the proton? Obtain the ratio of their speeds. (Electron mass $= 9.11 \times 10^{-31}$ kg, proton mass $= 1.67 \times 10^{-27}$ kg, $1 \text{eV} = 1.60 \times 10^{-19}$ J). **[NCERT]**

Sol. Here, $K_e = 10$ keV and $K_p = 100$ keV

$$m_e = 9.11 \times 10^{-31} \text{ kg}$$

and $m_p = 1.67 \times 10^{-27}$ kg

As $K = \frac{1}{2} mv^2$ or $v = \sqrt{\frac{2K}{m}}$,

Hence, $$\frac{v_e}{v_p} = \sqrt{\frac{K_e}{K_p} \times \frac{m_p}{m_e}}$$

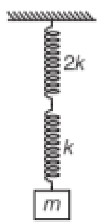
$$= \sqrt{\frac{10 \text{ keV}}{100 \text{ keV}} \times \frac{1.67 \times 10^{-27} \text{ kg}}{9.11 \times 10^{-31} \text{ kg}}}$$

$\Rightarrow$ $v_e = 13.54 \, v_p$

Thus, electron is travelling faster.

LONG ANSWER Type II Questions

24. A helicopter lifts a 72 kg astronaut 15 m vertically from the ocean by means of a cable. The acceleration of the astronaut is $\frac{g}{10}$. How much work is done on the astronaut by



(i) the force from the helicopter and
(ii) the gravitational force on him?
(iii) what are the kinetic energy and
(iv) the speed of the astronaut just before he reaches the helicopter? (Take, $g = 10 \text{ ms}^{-2}$)

Sol. Here, mass of astronaut $m = 72$ kg, vertical distance $h = 15$ m and acceleration of astronaut $a = \frac{g}{10}$.

(i) Force from the helicopter

$$F = mg + ma = mg + \frac{mg}{10} = \frac{11 \, mg}{10}$$

$\therefore$ Work done by force from the helicopter

$$W = F \times h = \frac{11 mg}{10} \times h$$

$$W = \frac{11 \times 72 \times 10 \times 15}{10} = 11880 \text{ J}$$

(ii) Work done by gravitational force

$$W' = (mg)(h)(\cos 180°)$$

$$W' = (mg)(h)(\cos 180°)$$
$$W' = -mgh = -72 \times 10 \times 15$$
$$= -10800 \text{ J}$$

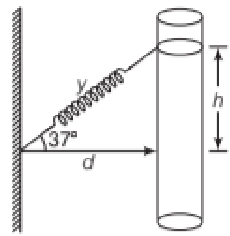
(iii) KE of astronaut K = network done on him

$$= W + W'$$
$$= (11880 - 10800) \text{ J} = +1080 \text{ J}$$

(iv) As KE $= \frac{1}{2} mv^2$, hence

$$v = \sqrt{\frac{2K}{m}} = \sqrt{\frac{2 \times 1080}{72}} = 5.48 \text{ ms}^{-1}$$

25. One end of a light spring of natural length d and spring constant k is fixed on a rigid wall and other end is fixed in a smooth ring of mass m as shown in figure. Initially, the spring is stretched such that it makes an angle of 37° with the horizontal.



Now, the ring is stretched from rest, find the speed of the ring when it will become horizontal, assume that ring slides on the vertical wall without friction.

When the spring is released, it slides down under gravity. The potential energy stored in the stretched spring and the potential energy of the ring released which appear as gain in kinetic energy of the ring.

Sol. We can first calculate the stretch l the spring over natural length in the initial situation.

$$\cos 37° = \frac{d}{y}$$

$$y = \frac{d}{\cos 37°}$$

$$= \frac{d}{4/5} = \frac{5d}{4} = 1.25 \, d$$

Similarly, $\tan 37° = \frac{h}{d} \Rightarrow \frac{3}{4} = \frac{h}{d}$

$$h = \frac{3}{4} d$$

Now, we can apply conservation of energy.

Total energy initially = Total energy when spring is horizontal.

$$\frac{1}{2} kl^2 + mgh = \frac{1}{2} mv^2$$

$$\frac{1}{2} k \, (0.25d)^2 + mg \, \frac{3}{4} d = \frac{1}{2} mv^2$$

$$\frac{3}{4} mg\, d + \frac{1}{2} k \frac{d^2}{16} = \frac{1}{2} mv^2$$

$$v = d\sqrt{\frac{3g}{2d} + \frac{k}{16m}}$$

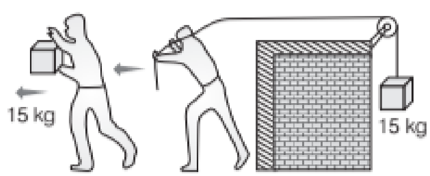
26. Answer the following

(i) The casing of a rocket in flight burns up due to friction. At whose expense is the heat energy required for burning obtained ? The rocket or the atmosphere.

(ii) Comets move around the Sun in highly elliptical orbits. The gravitational force on the comet due to the Sun is not normal to the comet's velocity in general. Yet the work done by the gravitational force over every complete orbit of the comet is zero. Why?

(iii) An artificial satellite orbiting the Earth in very thin atmosphere loses its energy gradually due to dissipation against atmospheric resistance, however small. Why then does its speed increase progressively as it comes closer and closer to the earth?

(iv) In Fig. (a) the man walks 2 m carrying a mass of 15 kg on his hands. In Fig. (b) he walks the same distance pulling the rope behind him. The rope goes over a pulley and a mass of 15 kg hangs at its other end. In which case is the work done greater?



(a) (b) **[NCERT]**

Sol. (i) Heat energy required for burning of casing of rocket comes from the rocket itself. As a result of work done against friction the kinetic energy of rocket continuously decreases and this work against friction reappears as heat energy.

(ii) The gravitational force is a conservative force, hence, work done by the gravitational force over one complete (closed) orbit of comet is zero.

(iii) As an artificial satellite gradually loses its energy due to dissipation against atmospheric resistance, its potential energy decreases rapidly. As a result, kinetic energy of satellite slightly increases i.e. its speed increases progressively.

(iv) In figure, the man carries the mass of 15 kg on his hands and walks 2 m. In this case, he is actually doing work against the friction force.

Friction force contribution by mass,

$$f = \mu N = \mu mg \times 15 \times 9.8 \text{ N}$$

and work done against friction,

$$W_1 = fs = \mu \times 15 \times 9.8 \times 2 = 294\, \mu\text{J}$$

In figure (ii), the tension in string, $T = mg = 15 \times 9.8$ N

Hence, force applied by man for pulling the rope

$$F = T = 15 \times 9.8 \text{ N}$$

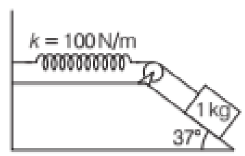
$\therefore$ Work done by man, $W_2 = Fs = 15 \times 9.8 \times 2 = 294$ J

and additional work has to be done against friction also.

Thus, it is clear that $W_2 > W_1$.

27. A 1 kg block situated on a rough incline is connected to a spring of spring constant 100 N m^{-1} as shown in figure. The block is released from rest with the spring in the unstretched position.

The block moves 10 cm down the incline before coming to rest. Find the coefficient of friction between the block and the incline. Assume that the spring has a negligible mass and the pulley is frictionless.



[Given, sin 37° = 0.6 and cos 37° = 0.8] **[NCERT]**

Sol. Here, force constant $k = 100$ Nm^{-1}, mass of block $m = 1$ kg, $\theta = 37°$ and distance moved by block $x = 10$ cm $= 0.1$ m

As shown in figure, the net accelerating force acting on block is $F = mg\sin\theta - f = mg\sin\theta - \mu N$

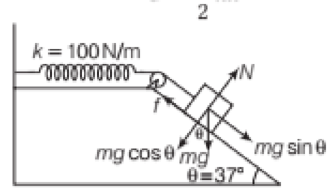
$$= mg\sin\theta - \mu\, mg\cos\theta$$

$\therefore$ Work done by the force F for motion of block

$$W = Fx = mg(\sin\theta - \mu\cos\theta)x$$

When the block stops, the work done is stored in the spring in the form of its potential energy

$$U = \frac{1}{2}kx^2$$



$$\therefore \quad mg(\sin\theta - \mu\cos\theta)x = \frac{1}{2}kx^2$$

$$\Rightarrow \quad \mu = \frac{1}{\cos\theta}\left[\sin\theta - \frac{kx}{2\, mg}\right]$$

Substituting the values, we get

$$\mu = \frac{1}{\cos 37°}\left[\sin 37° - \frac{100 \times 0.1}{2 \times 1 \times 10}\right]$$

$$= \frac{1}{0.8}[0.6 - 0.5] = 0.125$$

ASSESS YOUR TOPICAL UNDERSTANDING

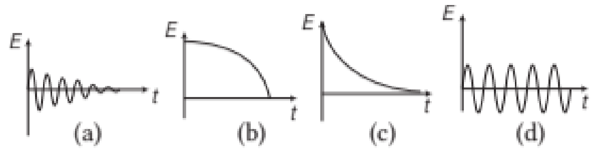
OBJECTIVE Type Questions

1. In daily life, intake of a human adult is 10^7 J, then average human consumption in a day is
(a) 2400 kcal (b) 1000 kcal
(c) 1200 kcal (d) 700 kcal

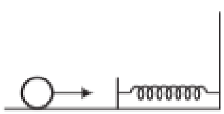
2. The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x?
(a) 10 J (b) 20 J
(c) 30 J (d) 40 J

3. The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy if they are stretched by the same force?
(a) 2 : 3 (b) 3 : 2
(c) 4 : 9 (d) 9 : 4

4. Which of the diagrams shown in figure represents variation of total mechanical energy of a pendulum oscillating in air as function of time? **[NCERT Exemplar]**



5. A mass of 0.5 kg moving with a speed of 1.5 ms^{-1} on a horizontal smooth surface, collides with a nearly weightless spring of spring constant $k = 50$ Nm^{-1}. The maximum compression of the spring would be



(a) 0.15 m (b) 0.12 m (c) 1.5 (d) 0.5 m

Answer

1. (a)	2. (c)	3. (b)	4. (c)	5. (a)

VERY SHORT ANSWER Type Questions

6. A spark is produced when two stones are struck against each other. Why?

7. Can energy be created or destroyed? Comment.

8. Name the largest and smallest practical unit of energy.

SHORT ANSWER Type Questions

9. Calculate the velocity of the bob of a simple pendulum at its mean position if it is able to rise to a vertical height of 10 cm. [$g = 9.8$ m/s^2] [**Ans.** 1.4 m/s]

10. What is the minimum amount of energy released in the annihilation of an electron-position pair? (Take, rest mass of electron-position = 9.11×10^{-31}kg and $c = 3 \times 10^8$ m/s) [**Ans.** 1.023 MeV]

11. What is meant by mass-energy equivalence? Discuss its significance in physics.

LONG ANSWER Type I Questions

12. Calculate the energy equivalent of 1 amu in MeV, taking 1 amu = 1.66×10^{-27} kg. [**Ans.** 933.75 MeV]

13. If 1000kg of water is heated from 0°C to 100°C. Calculate the increase in mass of water. [**Ans.** 4.66×10^{-9} kg]

14. Two bodies of unequal masses have same KE which one has greater linear momentum?

15. Define kinetic energy. Prove that KE associated with a mass m moving with velocity v is $\frac{1}{2}mv^2$.

LONG ANSWER Type II Questions

16. Explain, what is meant by potential energy of a spring? Obtain an expression for it and discuss the nature of its variation?

17. A stone of mass 0.4 kg is thrown vertically upward with a speed of 9.4 m/s. Find the potential and kinetic energies after half second. [**Ans.** 14.386 J and 4.802J]

18. If the linear momentum of a body increases by 20%, what will be the % increase in the kinetic energy of the body? [**Ans.** 44%]

19. A running man has half the kinetic energy that a boy of half his mass has. The man speeds up by 1.0 m/s and then same energy as the boy. What were the orignal speeds of the man and the boy? [**Ans.** 2.41 m/s, 4.828 m/s]

20. What is the work-energy theorem? Should it be named as work KE theorem? What exactly is work? Can it not be defined in terms of the expenses of energy? Explain your answer.

|TOPIC 3|
Power and Collision

POWER

Power of a person or machine is defined as the time rate at which work is done or energy is transferred by it.

If a person does work W in time t, then its average power is given by

$$\text{Average power } (P_{av}) = \text{Rate of doing work}$$
$$= \frac{\text{Work done}}{\text{Time taken}} = \frac{W}{t}$$

$$\boxed{\text{Average power, } P_{av} = \frac{W}{t}}$$

Thus, the average power of a force is defined as the ratio of the work, W to the total time t.

The **instantaneous power** of an agent is defined as the limiting value of the average power of an agent in a small time interval, when the time interval approaches to zero.

When work done by a force $\mathbf{F}$ for a small displacement $d\mathbf{r}$ is $dW = \mathbf{F} \cdot d\mathbf{r}$. Then, instantaneous power can be given as

i.e. $$P = \lim_{\Delta t \to 0} \frac{\Delta W}{\Delta t} = \frac{dW}{dt}$$

Now, $$dW = \mathbf{F} \cdot d\mathbf{r}$$

$\therefore$ $$P = \mathbf{F} \cdot \frac{d\mathbf{r}}{dt}$$

Again $\frac{d\mathbf{r}}{dt} = \mathbf{v}$, instantaneous velocity of an agent.

Therefore, $$P = \mathbf{F} \cdot \mathbf{v}$$

Thus, the power of an agent at any instant is equal to the dot product of its force and velocity vectors at that instant.

Dimensional Formula of Power

Power is a scalar quantity, because it is the ratio of two scalar quantities work (W) and time (t). The dimensional formula of power is $[ML^2T^{-3}]$.

Units of Power

The SI unit of power is watt (W). The power of an agent is one watt if it does work at the rate of 1 joule per second.

$$1 \text{ watt} = \frac{1 \text{ joule}}{1 \text{ second}} = 1 \text{ Js}^{-1}$$

Another popular units of power are kilowatt and horsepower.

1 kilowatt = 1000 watt or 1 kW = 10^3 W

1 horsepower = 746 watt or 1 hp = 746 W

This unit is used to describe the output of automobiles, motorbikes, engines, etc.

Note

- Kilowatt hour (kWh) or Board of Trade (BOT) is the commercial unit of electrical energy.
- Relation between kWh and joule.

$$1 \text{ kWh} = 1 \text{ kW} \times 1 \text{ h} = 1000 \text{ W} \times 1 \text{ h}$$
$$= 1000 \text{ Js}^{-1} \times 3600 \text{ s}$$

or $$1 \text{ kWh} = 3.6 \times 10^6 \text{ J}$$

EXAMPLE |1| Weighing a Man in a Lift

An elevator which can carry a maximum load of 1800 kg (elevator + passenger) is moving up with a constant speed of 2 ms^{-1}. The frictional force opposing the motion is 4000 N. Determine the minimum power delivered by the motor to the elevator in watts as well as in horsepower. [NCERT]

Sol. The downward force on the elevator is

$$F = mg + F_f$$

where, F_f is the frictional force

$$= 1800 \times 10 + 4000$$
$$= 22000 \text{ N}$$

The motor must supply enough power to balance this force. Hence, $P = Fv = 22000 \times 2 = 44000$ W

$$= \frac{44000}{746} \text{ hp} \simeq 59 \text{ hp}$$

EXAMPLE |2| Encounter with Two Cranes

A car of mass 2000 kg is lifted up a distance of 30 m by a crane in 1 min. A second crane does the same job in 2 min. Do the cranes consume the same or different amounts of fuel? What is the power supplied by each crane? Neglect power dissipation against friction. [NCERT]

Sol. Here, $m = 2000$ kg, $s = 30$ m

$$t_1 = 1 \text{ min} = 60 \text{ s}, \; t_2 = 2 \text{ min} = 120 \text{ s}$$

Work done by each crane,

$$W = Fs = mgs = 2000 \times 9.8 \times 30$$
$$= 5.88 \times 10^5 \text{ J}$$

As both the cranes do same amount of work, so both consume same amount of fuel.

Power supplied by first crane,

$$P_1 = \frac{W}{t_1} = \frac{5.88 \times 10^5}{60} = 9800 \text{ W}$$

Power supplied by second crane,

$$P_2 = \frac{W}{t_2} = \frac{5.88 \times 10^5}{120} = 4900 \text{ W}$$

EXAMPLE |3| Walking on Stairs

A boy of mass 40 kg walks up a flight of stairs to a vertical distance of 12 m, in a time interval of 40 s.

(i) At what rate is the boy doing work against the force of gravity?

(ii) If energy is transformed by the leg muscles of the students at the rate of 30 kJ every minute, what is the students power output?

Sol. (i) Given, $m = 40$ kg, $t = 40$ s

Energy = 30 kJ, power = ?

Student's power output = ?

Work done by the student against the force of gravity is equal to the gain in gravitational potential energy, so

Work done $(W) = mg\Delta h$

$$\text{Power} = \frac{\text{Work done}}{\text{Time taken } (\Delta t)}$$

$$P = \frac{W}{\Delta t} = \frac{40 \times 10 \times 12}{40} = 120 \text{ W}$$

(ii) Student's power output can be calculated as

$$\text{Power } (P) = \frac{\text{Energy transferred}}{\text{Time taken}}$$

$$P = 30 \text{ kJ/min} = \frac{30000 \text{ J}}{60 \text{ s}} = 500 \text{ W}$$

Mean Power of Projectile Motion on Horizontal Plane

As we know, mean power $= \dfrac{\text{Net gain in kinetic energy}}{\text{Total time of motion}}$

In projectile motion on horizontal plane, when a particle strikes the ground with the same speed with which it was projected, then kinetic energy of particle does not change. Hence, from initial point to final point, there is no gain in kinetic energy.

i.e. Mean power = 0

COLLISION

A collision is an isolated event in which two or more colliding bodies exert strong forces on each other for a relatively short time. For a collision to take place, the actual physical contact is not necessary.

e.g. In Rutherford's scattering experiment, an alpha particle moves towards nucleus of an atom get deflected by the electrostatic force of repulsion without actual physical contact with the nucleus. So, the alpha particle undergone collision with the nucleus.

Collision between particles have been divided into two types

	Elastic Collision	Inelastic Collision
(i)	A collision in which there is absolutely no loss of kinetic energy.	A collision in which there occurs some loss of kinetic energy.
(ii)	Forces involved during elastic collision must be conserved in nature.	Some or all forces involved during collision may be non-conservative in nature.
(iii)	The mechanical energy is not converted into heat, light, sound etc.	A part of the mechanical energy is converted into heat, light, sound, etc.
(iv)	e.g. Collision between subatomic particles, collision between glass balls, etc.	e.g. Collision between two vehicles, collision between a ball and floor.

Conservation of Linear Momentum in Collision

Consider two bodies 1 and 2 collide against each other. They exert mutual impulsive forces on each other during the collision time Δt. The changes produced in momenta of the two bodies will be

$$\Delta \mathbf{p}_1 = \mathbf{F}_{12}\, \Delta t \text{ and } \Delta \mathbf{p}_2 = \mathbf{F}_{21}\, \Delta t$$

where, $\mathbf{F}_{12}$ is the force exerted on body 1 by body 2 and $\mathbf{F}_{21}$ is the force exerted on body 2 by body 1.

According to Newton's third law of motion, we have

$$\mathbf{F}_{12} = -\mathbf{F}_{21}$$

$\therefore$ $\mathbf{F}_{12}\, \Delta t = -\mathbf{F}_{21}\, \Delta t$

or $\mathbf{F}_{12}\, \Delta t + \mathbf{F}_{21}\, \Delta t = 0$

$$\Delta \mathbf{p}_1 + \Delta \mathbf{p}_2 = 0$$

$$\Delta (\mathbf{p}_1 + \mathbf{p}_2) = 0$$

$$\boxed{\mathbf{p}_1 + \mathbf{p}_2 = \text{constant}}$$

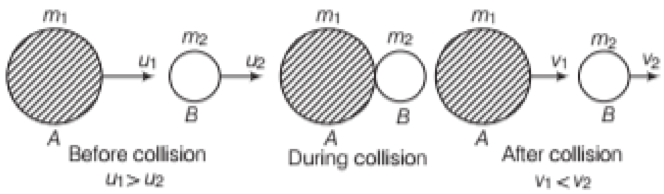
Hence, **total linear momentum is conserved at each instant during collision.**

Elastic Collision in One Dimension

It involves two bodies moving initially along the same straight line, striking against each other without loss of kinetic energy and continuing to move along the same straight line after collision.

Suppose two bodies A and B of masses m_1 and m_2 moving along the same straight line with velocities u_1 and u_2, respectively. Let $u_1 > u_2$.

After collision, bodies A and B moving with velocities v_1 and v_2 in the same direction such that $v_2 > v_1$ as shown in figure.



Elastic collision in one dimension

As linear momentum is conserved in any collision, we get

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 v_2 \qquad \ldots\text{(i)}$$

or $$m_1 u_1 - m_1 v_1 = m_2 v_2 - m_2 u_2$$

or $$m_1 (u_1 - v_1) = m_2 (v_2 - u_2) \qquad \ldots\text{(ii)}$$

Since, **KE is also conserved in an elastic collision**, we get

$$\frac{1}{2} m_1 u_1^2 + \frac{1}{2} m_2 u_2^2 = \frac{1}{2} m_1 v_1^2 + \frac{1}{2} m_2 v_2^2$$

or $$m_1 u_1^2 - m_1 v_1^2 = m_2 v_2^2 - m_2 u_2^2$$

or $$m_1 (u_1 + v_1)(u_1 - v_1) = m_2 (v_2 - u_2)(v_2 + u_2) \ldots\text{(iii)}$$

Dividing Eq. (iii) by Eq. (ii), we get

$$u_1 + v_1 = v_2 + u_2$$

or $$u_1 - u_2 = v_2 - v_1 \qquad \ldots\text{(iv)}$$

Hence, in one-dimensional elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision.

Velocities of the Bodies after the Collision

From Eq. (iv), we get

$$v_2 = u_1 - u_2 + v_1 \qquad \ldots\text{(v)}$$

Putting this value of v_2 in Eq. (i), we get

$$m_1 u_1 + m_2 u_2 = m_1 v_1 + m_2 (u_1 - u_2 + v_1)$$
$$= m_1 v_1 + m_2 u_1 - m_2 u_2 + m_2 v_1$$

or $$(m_1 - m_2) u_1 + 2 m_2 u_2 = (m_1 + m_2) v_1$$

Velocity of body A after collision,

$$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right) u_1 + \left(\frac{2m_2}{m_1 + m_2}\right) u_2 \qquad \ldots\text{(vi)}$$

Putting the value of v_1 in Eq. (v), we get

Velocity of body B after collision,

$$v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) u_2 + \left(\frac{2m_1}{m_1 + m_2}\right) u_1 \qquad \ldots\text{(vii)}$$

Eqs. (vi) and (vii) give the final velocities of the colliding bodies in terms of their initial velocities.

The two cases under the action of same and different masses can be considered as given below.

Case I When two bodies of equal masses collide.

i.e. $m_1 = m_2 = m$ say

From Eq. (vi), $v_1 = \dfrac{2mu_2}{2m} = u_2$

= velocity of body of mass m_2 before collision

From Eq. (vii), $v_2 = \dfrac{2mu_1}{2m} = u_1$

= velocity of body of mass m_1 before collision.

Hence, when two bodies of equal masses undergo perfectly elastic collision in one dimension, their velocities are just interchanged.

Case II When a light body collides against a massive stationary body.

Here, $m_1 << m_2$ and $u_2 = 0$

Neglecting m_1 in Eq. (vi), we get

$$v_1 = -\frac{m_2 u_1}{m_2} = -u_1$$

From Eq. (vii), $v_2 \simeq 0$.

Hence, when a light body collides against a massive body at rest, the light body rebounds after the collision with an equal and opposite velocity while the massive body practically remains at rest.

In an elastic collision, the kinetic energy conservation holds only after the collision is over. It does not hold during the short duration of actual collision.

At the time of collision, the two colliding objects are deformed and may be momentarily at rest with respect to each other.

If the initial velocities and final velocities of both the bodies are along the same straight line, then it is called a one-dimensional collision or head-on collision.

EXAMPLE |4| Head-on Collision

Two ball bearings of mass m each moving in opposite directions with equal speed collide head on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic. **[NCERT]**

Sol. Here, $m_1 = m_2 = m$ (say)

$u_1 = v, u_2 = -v$

We have taken $u_1 = v$ and $u_2 = -v$ because both the ball bearings moving with same velocity but in opposite directions.

As the collision is perfectly elastic, velocities after the collision will be

$$v_1 = \frac{m_1 - m_2}{m_1 + m_2} \cdot u_1 + \frac{2m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{m - m}{m + m} \cdot v + \frac{2m}{m + m}(-v) = 0 - v = -v$$

$$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_1}{m_1 + m_2} \cdot u_2$$

$$= \frac{2m}{m + m} \cdot v + \frac{m - m}{m + m} \cdot (-v) = v + 0 = v$$

Thus, the two balls bounce back with equal speed after the collision.

EXAMPLE |5| Slowing Down of Neutrons

In a nuclear reactor, a neutron of high speed (typically 10^7 ms^{-1}) must be slowed to 10^3 ms^{-1} so that it can have a high probability of interacting with isotope ${}^{325}_{92}U$ and causing it to fission.

Show that a neutron can lose most of its kinetic energy in an elastic collision with a light nucleus like deuterium or carbon which has a mass of only a few times the neutron mass. The material making up the light nuclei, usually heavy water (D_2O) or graphite is called a moderator. **[NCERT]**

Or

A body of mass M at rest is struck by a moving body of mass m. Prove that fraction of the initial KE of the mass m transferred to the struck body is $4mM/(m + M)^2$ in an elastic collision.

Sol. Here, m_1 = mass of neutron = m

m_2 = mass of target nucleus = M

$u_1 = u$ and $u_2 = 0$

Now, $$v_2 = \frac{2m_1}{m_1 + m_2} \cdot u_1 + \frac{m_2 - m_2}{m_1 + m_2} \cdot u_2$$

$$= \frac{2m}{m + M} \cdot u + 0 = \frac{2mu}{m + M}$$

Initial KE of mass m, $K_1 = \frac{1}{2} m_1 u_1^2 = \frac{1}{2} mu^2$

Final KE of mass M,

$$K_2 = \frac{1}{2} m_2 v_2^2 = \frac{1}{2} M \left(\frac{2\,mu}{m + M} \right)^2 = \frac{2\,Mm^2u^2}{(m + M)^2}$$

Fraction of the initial KE transferred,

$$f = \frac{K_2}{K_1} = \frac{2\,Mm^2\,u^2}{(m + M)^2} \times \frac{2}{mu^2} = \frac{4\,mM}{(m + M)^2}$$

(i) For deuterium, $M = 2m$, therefore

$$f = \frac{4m \times 2m}{(m + 2m)^2} = \frac{8}{9} \simeq 0.9$$

About 90% of the neutron's energy is transferred to deuterium.

(ii) For carbon, $M = 12m$, therefore

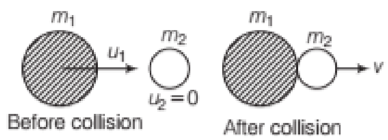
$$f = \frac{4m \times 12\,m}{(m + 12m)^2} = 0.284$$

About 28.4% of the neutron's energy is transferred to carbon.

Perfectly Inelastic Collision in One Dimension

When the two colliding bodies stick together and move as a single body with a common velocity after the collision, then the collision is **perfectly inelastic**.

As shown in figure, perfectly inelastic collision between two bodies of masses m_1 and m_2. The body of mass m_2 happens to the initially at rest ($u_2 = 0$). After the collision, the two bodies move together with common velocity v.



Perfectly inelastic collision

As the total linear momentum of the system remains constant, therefore $p_i = p_f$

i.e. $$m_1 u_1 + m_2 u_2 = (m_1 + m_2)\, v$$

$\therefore$ $$v = \frac{m_1 u_1}{m_1 + m_2} \qquad [\because u_2 = 0]$$

Total KE before collision,

$$E_1 = \frac{1}{2} m_1 u_1^2 \qquad \text{...(i)}$$

Total KE after collision,

$$E_2 = \frac{1}{2}(m_1 + m_2) v^2 \qquad \text{...(ii)}$$

On putting the value of v in Eq. (ii), we get

$$E_2 = \frac{1}{2}(m_1 + m_2) \left[\frac{m_1 u_1}{m_1 + m_2} \right]^2 = \frac{1}{2} \frac{m_1^2 u_1^2}{(m_1 + m_2)}$$

$$\text{Loss of KE} = E_1 - E_2 = \frac{1}{2} m_1 u_1^2 - \frac{m_1^2 u_1^2}{2\,(m_1 + m_2)}$$

$$\Delta\,\text{KE} = \frac{m_1^2 u_1^2 + m_1 m_2\, u_1^2 - m_1^2 u_1^2}{2\,(m_1 + m_2)} = \frac{m_1 m_2 u_1^2}{2\,(m_1 + m_2)}$$

$\therefore$ **ΔKE is a positive quantity.**

Therefore, kinetic energy is lost mainly in the form of light, sound and heat.

EXAMPLE |6| A Railway Carriage got an Accident

A railway carriage of mass 9000kg moving with a speed of 36km/h collides with a stationary carriage of the same mass. After the collision, the carriage get coupled and move together. What is their common speed after collision? What type of collision is this?

Sol. Given, $m_1 = 9000$ kg, $u_1 = 36$ km/h $= 36 \times \frac{5}{18} = 10$ m/s

$$m_2 = 9000 \text{ kg}, u_2 = 0, v_1 = v_2 = v$$

By conservation of momentum

$$m_1u_1 + m_2u_2 = (m_1 + m_2)v$$

$$9000 \times 10 + 9000 \times 0 = (9000 + 9000)v$$

or $$v = \frac{90000}{18000} = 5 \text{ m/s}$$

$$\text{Total KE before collision} = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$$

$$= \frac{1}{2} \times 9000 \times 10 \times 10 + 0$$

$$= 450000 \text{ J}$$

$$\text{Total KE after collision} = \frac{1}{2}(m_1 + m_2)v^2$$

$$= \frac{1}{2} \times 2 \times 9000 \times (5)^2$$

$$= 225000 \text{ J}$$

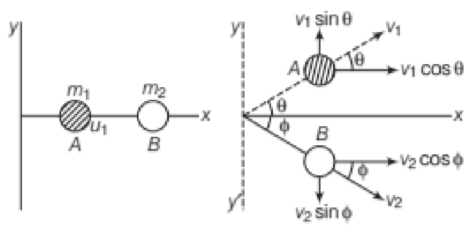
Thus, total KE after collision < Total KE before collision

Hence, the collision is inelastic.

Elastic Collision in Two Dimensions or Oblique Collision

Consider two objects *A* and *B* of masses m_1 and m_2 kept on the *x*-axis as shown in figure. Initially, the object *B* is at rest and *A* moves towards *B* with a speed u_1. If the collision is not head-on (the force during the collision is not along the initial velocity), the objects move along different lines.

Suppose the object *A* moves with a velocity $\mathbf{v}_1$ making an angle θ with the *x*-axis and the object *B* moves with a velocity $\mathbf{v}_2$ making an angle ϕ with the same axis as shown in figure.



Elastic collision in two dimensions

Also, suppose $\mathbf{v}_1$ and $\mathbf{v}_2$ lie in *xy*-plane. Using conservation of momentum in *x* and *y*-directions, we get

$$m_1u_1 = m_1v_1 \cos\theta + m_2v_2 \cos\phi$$

$$[\because u_2 = 0] \quad \ldots\text{(i)}$$

$\because$ The initial momentum of m_1 and m_2 along *y*-axis is zero, then

$$0 = m_1v_1 \sin\theta - m_2v_2 \sin\phi \quad \ldots\text{(ii)}$$

If the collision is elastic, the final kinetic energy is equal to the initial kinetic energy. Thus,

$$\frac{1}{2}m_1u_1^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

or $$m_1u_1^2 = m_1v_1^2 + m_2v_2^2 \quad \ldots\text{(iii)}$$

The four unknown quantities v_1, v_2, θ and ϕ cannot be calculated by using the three equations (i), (ii) and (iii).

But by measuring one of the four unknowns, say θ experimentally. The value of other three unknowns can be solved.

The three cases can be considered as given below.

Case I Glancing collision In a glancing collision, the incident particle does not lose any kinetic energy and is scattered almost undeflected. Thus, for such collision, when $\theta \simeq 0°$ and $\phi \simeq 90°$

From Eqs. (i) and (ii), we get

$$u_1 = v_1 \text{ and } v_2 = 0$$

$$\text{KE of the target particle} = \frac{1}{2}m_2v_2^2 = 0$$

Case II Head-on collision In this type of collision, the target particle moves in the direction of the incident particle, i.e. $\phi = 0°$. Then, Eqs. (i) and (ii), become

$$m_1u_1 = m_1v_1 \cos\theta + m_2v_2 \text{ and } 0 = m_1v_1 \sin\theta$$

Eq. (iii) for the kinetic energy remains unchanged.

Case III Elastic collision of two identical particles

When two particles of same mass undergo perfectly elastic collision in two dimensions, i.e. $m_1 = m_2$.

Let us take $u_1 = u$

From Eq. (iii), $v_1^2 + v_2^2 = u^2 \quad \ldots\text{(iv)}$

From Eq.(i) $v_1 \cos\theta + v_2 \cos\phi = u \quad \ldots\text{(v)}$

From Eq. (ii), $v_1 \sin\theta - v_2 \sin\phi = 0 \quad \ldots\text{(vi)}$

Using Eq. (v), we obtain from Eq. (iv)

$$v_1^2 + v_2^2 = (v_1 \cos\theta + v_2 \cos\phi)^2 = v_1^2 \cos^2\theta + v_2^2 \cos^2\phi + 2v_1v_2 \cos\theta \cos\phi$$

$$\Rightarrow v_1^2 (1 - \cos^2\theta) + v_2^2 (1 - \cos^2\phi) = 2v_1v_2 \cos\theta \cos\phi$$

$$v_1^2 \sin^2\theta + v_2^2 \sin^2\phi = 2v_1 v_2 \cos\theta \cos\phi \quad \text{...(vii)}$$

From Eq. (vi), $v_2 \sin\phi = v_1 \sin\theta$

Put in Eq. (vii), $2v_1^2 \sin^2\theta = 2v_1 v_2 \cos\theta \cos\phi$

$$\text{or} \cos\theta = \frac{2v_1^2 \sin^2\theta}{2v_1 v_2 \cos\phi} = \left[\frac{v_1}{v_2}\right] \times \frac{\sin^2\theta}{\cos\phi} \quad \text{...(viii)}$$

We know $\cos(\theta + \phi) = \cos\theta \cos\phi - \sin\theta \sin\phi$

Using Eq. (viii) and Eq. (vi), we get

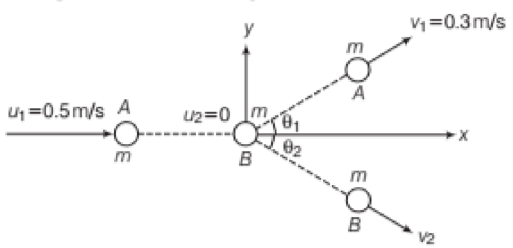
$$\cos(\theta + \phi) = \frac{v_1 \sin^2\theta}{v_2 \cos\phi} \cos\phi - \frac{v_1}{v_2} \sin^2\theta = 0$$

$\therefore \quad \theta + \phi = 90°$

Hence, we conclude that in a perfectly elastic collision, when a moving particle of mass m collides elastically in two dimensions with another particle of mass m, after collision the two particles will move at right angle to each other.

EXAMPLE |7| Collision at an Intersection

Consider the collision depicted in figure to be between two billiard balls with equal masses $m_1 = m_2$. The first ball is called the cue while the second ball is called the target. The billiard player wants to sink the target ball in a corner pocket, which is at an angle $\theta_2 = 37°$. Assume that the collision is elastic and that friction and rotational motion are not important. Obtain θ_1. **[NCERT]**



Sol. By conservation of momentum, we have

$$m\mathbf{u}_1 + 0 = m\mathbf{v}_1 + m\mathbf{v}_2$$

or $\quad \mathbf{u}_1 = \mathbf{v}_1 + \mathbf{v}_2 \quad$ ...(i)

By conservation of energy, we have

$$\frac{1}{2} m u_1^2 = \frac{1}{2} m v_1^2 + \frac{1}{2} m v_2^2$$

or $\quad u_1^2 = v_1^2 + v_2^2 \quad$ ...(ii)

From Eq. (i), we have

$$\mathbf{u}_1 \cdot \mathbf{u}_1 = (\mathbf{v}_1 + \mathbf{v}_2) \cdot (\mathbf{v}_1 + \mathbf{v}_2)$$
$$= \mathbf{v}_1 \cdot \mathbf{v}_1 + \mathbf{v}_1 \cdot \mathbf{v}_2 + \mathbf{v}_2 \cdot \mathbf{v}_1 + \mathbf{v}_2 \cdot \mathbf{v}_2$$

or $\quad u_1^2 = v_1^2 + v_2^2 + 2\mathbf{v}_1 \cdot \mathbf{v}_2$

or $\quad u_1^2 = u_1^2 + 2\mathbf{v}_1 \cdot \mathbf{v}_2 \quad$ [using Eq. (ii)]

or $\quad \mathbf{v}_1 \cdot \mathbf{v}_2 = 0$

Thus, the angle between $\mathbf{v}_1$ and $\mathbf{v}_2$ is 90°.

or $\quad \theta_1 + \theta_2 = 90°$

$\therefore \quad \theta_1 = 90° - \theta_2$

$= 90 - 37° = 53°$

Inelastic Collision in Two Dimensions

When two bodies travelling initially along the same straight line collide involving some loss of kinetic energy and move after collision along different directions in a plane.

Steps to be used for Solving Problems based on Two Dimensional Collision

Step I Firstly find out both x and y-coordinates. Its convenient to have either the x-axis or y-axis coincide with the direction of one of the initial velocities.

Step II Draw a diagram, labelling velocity vectors and masses.

Step III Write a separate conservation of momentum equation for each of the x and y-axes. In each case, total initial momentum in a given direction equal to the final momentum in that direction.

Step IV If the collision is in elastic, then write a general expression for the total energy before and after the collision.

Equate both the equations to get unknown value.

Step V There are two equations for inelastic collisions and three for elastic collision.

EXAMPLE |8| Inelastic Collision between Two Balls

A ball moving with a speed of 9 m/s strikes an identical ball at rest, such that after collision, the direction of each ball makes an angle of 30° with the original line of motion. Find the speed of the two balls after collision.

Sol. Given, $\quad m_1 = m_2 = m,\ u_1 = 9$ m/s, $u_2 = 0$

$\theta_1 = \theta_2 = 30°,\ v_1 = ?,\ v_2 = ?$

According to principle of conservation of linear momentum.

$$m_1 u_1 + m_2 u_2 = m_1 v_1 \cos\theta_1 + m_2 v_2 \cos\theta_2$$

Along the direction of motion in x-axis

$$m \times 9 + 0 = m v_1 \cos 30° + m v_2 \cos 30°$$

$$9 = v_1 \frac{\sqrt{3}}{2} + v_2 \frac{\sqrt{3}}{2} = \frac{(v_1 + v_2)\sqrt{3}}{2}$$

$$v_1 + v_2 = \frac{18}{\sqrt{3}} \quad \text{...(i)}$$

Along a direction of motion in y-axis

$$0 + 0 = m_1 v_1 \sin\theta_1 - m_2 v_2 \sin\theta_2$$
$$m_1 v_1 \sin\theta_1 = m_2 v_2 \sin\theta_2$$

$$\Rightarrow \quad m v_1 \sin 30° = m v_2 \sin 30°$$

$$\Rightarrow \qquad v_1 = v_2 \qquad \text{...(ii)}$$

Substituting value from Eq. (ii) in Eq. (i), then find out values of v_1 and v_2

$$2v_1 = \frac{18}{\sqrt{3}}$$

$$\Rightarrow \qquad v_1 = \frac{9}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$$

$$= \frac{9\sqrt{3}}{3} = 3\sqrt{3} \text{ m/s}$$

Hence, two balls move with same velocity $= 3\sqrt{3}$ m/s after collision.

Coefficient of Restitution or Coefficient of Resilience

It is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by e.

$$e = \frac{\text{Relative velocity of separation (after collision)}}{\text{Relative velocity of approach (before collision)}}$$

$$e = \frac{|v_2 - v_1|}{|u_1 - u_2|} = \frac{v_2 - v_1}{u_1 - u_2}$$

$$\boxed{\text{Coefficient of resilience, } e = \frac{v_2 - v_1}{u_1 - u_2}}$$

where u_1, u_2 are velocities of two bodies before collision and v_1, v_2 are their respective velocities after collision.

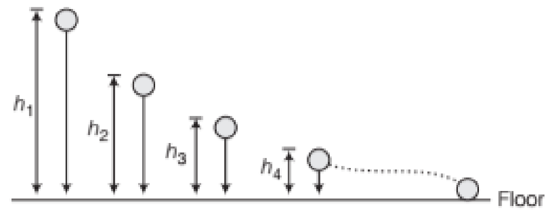
- The coefficient of restitution gives a measure of the degree of restitution of a collision. The value of e depends on the materials of the colliding bodies. e.g. for two glass balls, $e = 0.95$ and for the lead balls, $e = 0.20$.
- For a perfectly elastic collision, relative velocity of separation after collision is equal to relative velocity of approach before collision
  $\therefore \qquad e = 1$
  For a perfectly inelastic collision, relative velocity of separation after collision = 0 i.e. = 0
- For all other collisions, e lies between 0 and 1, i.e. $0<e<1$

Rebouncing of a Ball

If a ball is dropped from certain height h, then it rebounds from a floor as shown in figure.

So, height attained after n impacts is $h_n = e^{2n} h$. Then, total distance travelled before the body comes to rest is

$$s = h\left(\frac{1+e^2}{1-e^2}\right)$$, where e is coefficient of restitution.



If we add impact heights at every instance, then it forms geometric progression.

Different Types of Collisions

Collision	Kinetic energy	Coefficient of restitution	Main domain
Elastic	Conserved	$e = 1$	Between atomic particles
Inelastic	Not conserved	$0 < e < 1$	Between ordinary objects
Perfectly inelastic	Maximum loss of KE	$e = 0$	During shooting
Super elastic	KE increases	$e > 1$	In explosions

EXAMPLE |9| Momentum of a Bob

A sphere of mass m moving with a velocity u hits another stationary sphere of same mass. What will be the ratio of the velocities of two spheres after the collision. Take e as the coefficient of restitution.

Sol. Here, $u_1 = u, u_2 = 0$

$$\therefore \qquad e = \frac{v_2 - v_1}{u_1 - u_2} = \frac{v_2 - v_1}{u - 0}$$

or $\qquad v_2 - v_1 = e\,u \qquad$ ...(i)

According to the law of conservation of momentum, we have

$$mu + m \times 0 = mv_1 + mv_2$$

or $\qquad v_1 + v_2 = u \qquad$ ...(ii)

On adding Eqs. (i) and (ii), we get

$$2v_2 = u + eu = u(1+e)$$

or $\qquad v_2 = \frac{u(1+e)}{2} \qquad$ ...(iii)

Substituting the value of v_2 in Eq. (ii), we have

$$v_1 = u - v_2$$

$$= u - \frac{u(1+e)}{2}$$

$$= \frac{u(1-e)}{2} \qquad \text{...(iv)}$$

Dividing Eq. (iii) by Eq. (iv), we get

$$\frac{v_2}{v_1} = \frac{1+e}{1-e}$$

TOPIC PRACTICE 3

OBJECTIVE Type Questions

1. A particle is acted by a constant power. Then, which of the following physical quantity remains constant?
(a) Speed
(b) Rate of change of acceleration
(c) Kinetic energy
(d) Rate of change of kinetic energy

Sol. (d) By definition, $P = \frac{dW}{dt}$

$\because$ Work done = Kinetic energy

$\Rightarrow P = \frac{dW}{dt} = \frac{d(\text{KE})}{dt} = \text{constant}$

2. The power (P) of an engine lifting a mass of 100 kg upto a height of 10 m in 1 min is
(a) 162.3 W (b) 163.3 W
(c) 164.3 W (d) 165 W

Sol. (b) Power $= \frac{\text{Work}}{\text{Time}} = \frac{mgh}{t}$

Here, $m = 100$ kg, $h = 10$ m and $t = 1$ min $= 60$ s.

$\therefore P = \frac{100 \times 9.8 \times 10}{60} = 163.3 \text{ W}$

3. A man does a given amount of work in 10 s. Another man does the same amount of work in 20 s. The ratio of the output power of the first man to that of second man is
(a) 1 (b) 1 : 2 (c) 2:1 (d) 3 :1

Sol. (c) Since, $P = \frac{W}{t}$

So, if W is constant, then $P \propto \frac{1}{t}$

i.e. $\frac{P_1}{P_2} = \frac{t_2}{t_1} = \frac{20}{10} \Rightarrow \frac{P_1}{P_2} = \frac{2}{1}$ or $P_1 : P_2 = 2:1$

4. A particle of mass m_1 moves with velocity v_1 collides with another particle at rest of equal mass. The velocity of second particle after the elastic collision is
(a) $2v_1$ (b) v_1 (c) $-v_1$ (d) 0

Sol. (b) Given, mass $m_1 = m_2 = m$ and velocity, $v = v_1$

For elastic collision, $v_2 = \left(\frac{m_2 - m_1}{m_1 + m_2}\right) v_2 + \frac{2m_1 v_1}{m_1 + m_2}$

After putting given values, we will get

$$v_2 = \frac{2m_1 v_1}{2m_1} \Rightarrow v_2 = v_1$$

5. A body of mass 5 kg is thrown vertically up with a kinetic energy of 490 J. The height at which the kinetic energy of the body becomes half of the original value is
(a) 12.5 m (b) 10 m (c) 2.5 m (d) 5 m

Sol. (d) According to the law of conservation of energy,

$$\frac{1}{2}mv^2 = \frac{1}{2}\left(\frac{1}{2}mv^2\right) + mgh$$

$$\Rightarrow \quad 490 = 245 + 5 \times 9.8 \times h, \quad h = \frac{245}{49} = 5\text{m}$$

VERY SHORT ANSWER Type Questions

6. Calculate the power of an electric engine which can lift 20 tonne of coal per hour from a mine 180 m deep.

Sol. Power $= \frac{\text{Work done}}{\text{Time taken}} = \frac{mgh}{t} = \frac{20 \times 1000 \times 9.8 \times 180}{60 \times 60}$

$= 9800 \text{ W} = 9.8 \text{ kW}$

7. Is collision between two particles possible even without any physical contact between them?

Sol. Yes, in atomic and subatomic particles collision without any physical contact between the colliding particles is taking place e.g. Rutherford's alpha particles scattering.

8. In what type of collision, maximum kinetic energy is transferred?

Sol. Maximum kinetic energy is transferred when two bodies of equal mass collide.

9. If the momentum and total energy is conserved, then define the collision is occurred?

Sol. Collision in which momentum and total energy remained conserved and total kinetic energy of the colliding particles remain constant both before and after the collision, is called **elastic collision**.

10. In which of the two types of collision i.e. elastic or inelastic, the momentum is conserved? What about KE?

Sol. Momentum is conserved in both the types of collisions, but KE is conserved only in elastic collision.

11. Which physical terms remain conserved in an inelastic collision?

Sol. In an inelastic collision, total momentum as well as total energy remain conserved.

12. Is the total linear momentum conserved during the short time of an elastic collision of two balls ?

Sol. During the short interval of an elastic collision, total linear momentum is conserved.

13. What is the loss in kinetic energy after collision, if the target body is initially at rest?

Sol. Loss in kinetic energy on collision is $\frac{1}{2}\left(\frac{m_1 m_2}{m_1 + m_2}\right)u^2$, where u is the initial velocity.

SHORT ANSWER Type Questions

14. A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal, and rebounds with the same speed. Is momentum conserved in the collision? Is the collision elastic or inelastic? [NCERT]

Sol. Yes, momentum remains conserved in the collision.
To check whether the collision is elastic or inelastic, we consider the kinetic energy of the molecule.
We find that the initial kinetic energy $\left(\frac{1}{2}mu^2\right)$ is the same as final KE. $\left(\frac{1}{2}mv^2\right)$ of the molecule as
$u = v = 200$ m / s i.e. thus the collision is elastic collision.

15. Two ball bearings of mass m each moving in opposite directions with equal speed v collide head-on with each other. Predict the outcome of the collision, assuming it to be perfectly elastic.

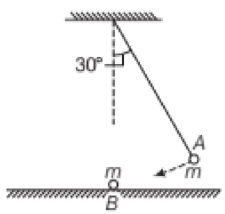
Sol. Here, $M_1 = M_2 = m, u_1 = v$ and $u_2 = -v$

Now,
$$v_1 = \frac{(M_1 - M_2)u_1 + 2M_2 u_2}{M_1 + M_2} = \frac{(m-m)v + 2m(-v)}{m+m} = -v$$

and
$$v_2 = \frac{(M_2 - M_1)u_2 + 2M_1 u_1}{M_1 + M_2} = \frac{(m-m)(-v) + 2mv}{m+m} = v$$

After collision, the two ball bearings will move with same speed but their directions of motion will be reversed.

16. The bob A of a pendulum released from 30° to the vertical hits another bob B of the same mass at rest on a table as shown in figure.



How high does the bob A rise after the collision? Neglect the size of the bobs and assume the collision to be elastic. [NCERT]

Sol. When the pendulum bob A reaches the position B, its velocity is horizontal and it strikes the mass m placed at B. Since, the collision is one-dimensional and elastic, the bob exchange their velocities being of the same mass. The bob A does not rise at all and the bob B begins to move with the velocity of A.

17. A particle of mass 1 kg moving with a velocity $\mathbf{v}_1 = (3\hat{i} - 2\hat{j})$ m/s experience a perfectly inelastic collision with another particle of mass 2 kg having velocity $\mathbf{v}_2 = (4\hat{j} - 6\hat{k})$ m/s. Find the velocity and speed of the particle formed.

Sol. Given, $m_1 = 1$ kg, $\mathbf{v}_1 = (3\hat{\mathbf{i}} - 2\hat{\mathbf{j}})$ m/s, $m_2 = 2$ kg and
$$\mathbf{v}_2 = (4\hat{\mathbf{j}} - 6\hat{\mathbf{k}})\text{ m/s}$$
When two particles experience a perfectly inelastic collision. They stick together and move with a common velocity v given by
$$\mathbf{v} = \frac{m_1\mathbf{v}_1 + m_2\mathbf{v}_2}{m_1 + m_2} = \frac{1(3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}) + 2(4\hat{\mathbf{j}} - 6\hat{\mathbf{k}})}{1+2}$$
$$= (\hat{\mathbf{i}} + 2\hat{\mathbf{j}} - 4\hat{\mathbf{k}})\text{ m/s}$$
Speed of combined particle $v = \sqrt{1^2 + (2)^2 + (-4)^2} = \sqrt{21}$ m/s

18. Calculate the power of a motor which is capable of raising of water in 5 min from a well 120 m deep.

Sol. Here, the volume of water raised $V = 2000$ L
Density of water, $\rho = 1$ kg/L
$\therefore$ Mass of water raised, $m = V\rho = 2000 \times 1 = 2000$ kg
$$\text{Power, } P = \frac{W}{t} = \frac{mgh}{t} = \frac{2000 \times 9.8 \times 120}{5 \times 60} = 7840 \text{ W}$$
$= 7.840$ kW [1kW = 1000 W]

LONG ANSWER Type I Questions

19. A pump on the ground floor of a building can pump up water to fill a tank of volume 30 m^3 in 15 min. If the tank is 40 m above the ground, and the efficiency of the pump is 30%, how much electric power is consumed by the pump? [NCERT]

Sol. Here, volume of water lifted V = volume of tank $= 30\text{ m}^3$, time t = 15 min = 900 s,
Height of tank, $h = 40$ m and efficiency of motor, $\eta = 30\%$
$\therefore$ Output of motor = work done to raise water
$$= mgh = V\rho gh$$
$$\therefore \text{ Output power} = \frac{V\rho gh}{t} = \frac{30 \times 10^3 \times 9.8 \times 40}{900}$$
$$= 1.307 \times 10^4 \text{ W}$$
$$\therefore \quad \text{Input power} = \frac{\text{Output power}}{\eta} = + \frac{1.307 \times 10^4}{\frac{30}{100}}$$
$$= \frac{1.307 \times 10^4 \times 100}{30} = 4.357 \times 10^4 \text{ W}$$
$= 43.57$ kW [1kW=1000W]

20. The blades of a windmill sweep out a circlef area A. (i) If the wind flows at a velocity v perpendicular to the circle, what is the mass of the air passing through it in time t? (ii) What is the kinetic energy of the air? (iii) Assume that the windmill converts 25% of the wind's energy into electrical energy, and that $A = 30\ m^2$, $v = 36$ km/h and the density of the air is 1.2 kg m^{-3}. What is the electrical power produced?

Sol. (i) Area swept by blades of windmill = A and wind velocity = v

$\therefore$ Volume of air passing per unit time = Av

$\therefore$ Mass of air passing per unit time = $Av\rho$

and mass of air passing in time t, $M = Av\rho t$

(ii) KE of said quantity of air, $K = \frac{1}{2} Mv^2 = \frac{1}{2} A\rho t v^3$

(iii) If efficiency of windmill be 25%, then

Output electrical power = 25% of input power

$$= \frac{25}{100} \times \frac{1}{2} A\rho v^3$$

As $A = 30\ m^2$, $v = 36\ km/h = 36 \times \frac{5}{18}$ m/s

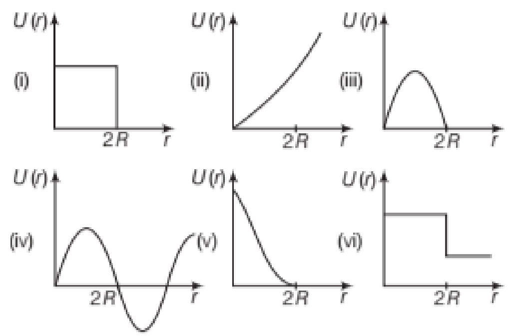
$= 10$ m/s and $\rho = 1.2\ kgm^{-3}$

$\therefore$ Output electrical power $= \frac{25}{100} \times \frac{1}{2} \times 30 \times 1.2 \times (10)^3$

$= 4500$ W$= 4.5$ kW

[1 kW = 1000 W]

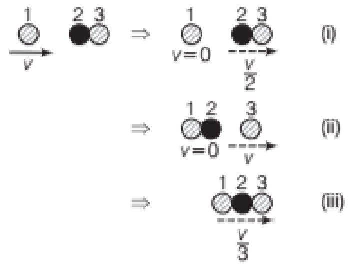
21. Which of the following potential energy curves in figure cannot possibly describe the elastic collision of two billiard balls? Here, r is the distance between centres of the balls. [NCERT]



Sol. When two billiard balls collide then distance between their centres is $2R$. Due to impact of collision there is small temporary deformation of balls. In this process, KE of ball is gradually reduced to zero and converted into elastic potential energy of balls.

When KE is zero, the balls regain their original configuration due to elasticity. The phenomenon can be successfully explained only by potential energy curve number (v), because here as $r < 2R$, the potential energy function $U(r)$ is increasing gradually on decreasing value of r.

22. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed v. If the collision is elastic, which of the following (figure) is a possible result after collision? [NCERT]



Sol. We know that, when two bodies of same mass collide, they interchange their velocities. When first ball collides with the combinations of balls 2 and 3, the first ball comes to rest while ball number 2 moves with velocity v again it collides with ball number 3 and hence ball number 2 comes to rest while ball number 3 moves with velocity v.

Hence, the situation shown in Fig. (ii) is the correct result of collision.

23. A bullet of mass 0.012 kg and horizontal speed 70 ms^{-1} strikes a block of wood of mass 0.4 kg and instantly comes to rest with respect to the block. The block is suspended from the ceilling by means of thin wires. Calculate the height to which the block rises. Also, estimate the amount of heat produced to the block. [NCERT]

Sol. Here, mass of bullet $m = 0.012$ kg, initial speed of bullet $u = 70\ ms^{-1}$, mass of wood block $M = 0.4$ kg.

As on collision, the bullet comes to rest w.r.t. block, it means that after collision bullet and block are moving with a common speed v.

From conservation law of momentum $mu = (M + m)\, v$

$$\Rightarrow \quad v = \frac{mu}{(M + m)} = \frac{0.012 \times 70}{(0.4 + 0.012)}$$

$$= 2.04\ m\ s^{-1}$$

If the block now rises to a maximum height of h, then using conservation law of mechanical energy, we have

$$\frac{1}{2}(M + m)\, v^2 - 0 = (M + m)\, gh$$

$$\Rightarrow \quad h = \frac{v^2}{2g} = \frac{(2.04)^2}{2 \times 9.8} = 0.212\ m = 21.2\ cm$$

24. A trolley of mass 200 kg moves with a uniform speed of 36 km/h on a frictionless track. A child of mass 20 kg runs on the trolley from one end to the other (10 m away) with a speed of 4 ms^{-1} relative to the trolley in a direction opposite to its motion, and jumps out of the trolley. What is the final speed of the trolley? How much has the trolley moved from the time the child begins to run? [NCERT]

Sol. Let there be an observer travelling parallel to the trolley with the same speed. He will observe the initial momentum of the trolley of mass M and child of mass m as zero. When the child jumps in opposite direction, he will observe the increase in the velocity of the trolley by Δv. Let u be the velocity of the child. He will observe child landing at velocity $(u - \Delta v)$.

Therefore, final momentum $= M\Delta v - m(u - \Delta v)$

From the law of conservation of momentum, we have

$$M\Delta v - m(u - \Delta v) = 0 \Rightarrow \Delta v = \frac{mu}{M+m}$$

Putting various values, we have

$$\Delta v = \frac{4 \times 20}{20 + 220} = 0.33 \text{ ms}^{-1}$$

∴ Final speed of trolley is 10.36 ms^{-1}

The child take 2.5 s to run on the trolley.

Therefore, the trolley moves a distance

$$= 2.5 \times 10.36 \text{ m} = 25.9 \text{ m}$$

25. A family uses 8 kW of power. (i) Direct solar energy is incident on the horizontal surface at an average rate of 200 W per square metre. If 20% of this energy can be converted to useful electrical energy, how large an area is needed to supply 8 kW? (ii) Compare this area to that of the roof of a typical house. [NCERT]

Sol. (i) Power used by family, $P = 8 \text{ kW} = 8000 \text{ W}$

As only 20% of solar energy can be converted to useful electrical energy, hence power to be supplied by solar energy $\frac{8000 \text{ W}}{20\%} = 40000 \text{ W}$

As solar energy is incident at a rate of 200 Wm^{-2}, hence the area needed

$$A = \frac{40000 \text{ W}}{200 \text{ Wm}^{-2}} = 200 \text{ m}^2$$

(ii) The area needed is comparable to roof area of a large sized house.

26. A synchronous motor is used to lift an elevator and its load of 1500 kg to a height of 20 m. The time taken for job is 20 s. What is work done? What is the rate at which work is done?

If the efficiency of the motor is 75%, at which rate is the energy supplied to the motor?

Sol. Given, mass, $m = 1500$ kg, $h = 20$ m, $\eta = 75\%$, $t = 20$ s

Work done, $W = mgh = 1500 \times 9.8 \times 20$

$$= 2.94 \times 10^5 \text{ J}$$

$$\text{Rate of doing work} = \frac{W}{t} = \frac{2.94 \times 10^5}{20} = 1.47 \times 10^4 \text{ W}$$

As, efficiency, $\eta = \frac{\text{Output power}}{\text{Input power}}$

$$\frac{75}{100} = \frac{1.47 \times 10^4}{\text{Input power}}$$

Input power or the rate at which energy is supplied

$$= \frac{1.47 \times 10^4 \times 100}{75} = 1.96 \times 10^4 \text{ W}$$

LONG ANSWER Type II Questions

27. State if each of the following statements is true or false. Give reasons for your answer.

(i) In an elastic collision of two bodies, the momentum and energy of each body is conserved.

(ii) Total energy of a system is always conserved, no matter what internal and external forces on the body are present?

(iii) Work done in the motion of a body over a closed loop is zero for every force in nature.

(iv) In an inelastic collision, the final kinetic energy is always less than the initial kinetic energy of the system. [NCERT]

Sol. (i) False, in elastic collision the linear momentum and kinetic energy of the system as a whole is conserved, but momentum and kinetic energy of each of individual body change.

(ii) False, internal as well as external forces can change the kinetic energy. Again forces if conservative may change the potential energy of a system.

(iii) False, for non-conservative force the work done over a closed loop is not zero.

(iv) It is usually true but not always true. As an example in the explosion of a cracker final kinetic energy is greater than the initial kinetic energy.

Again final kinetic energy of gun-bullet system after firing is more than initial kinetic energy before collision.

28. Answer carefully with reasons

(i) In an elastic collision of two billiard balls, is the total kinetic energy conserved during the short time of collision of the balls i.e. when they are in contact? [NCERT]

(ii) Is the total linear momentum conserved during the short time of an elastic collision of two balls?

(iii) What are the answer to (i) and (ii) for an inelastic collision?

(iv) If the potential energy of two billiard balls depends only on the separation distance between their centres, is the collision elastic or inelastic? (Note : We are talking here of potential energy corresponding to the force during collision, not gravitational potential energy.)

Sol. (i) No, the total kinetic energy, does not remain conserved during the short time when two billiard balls are in contact. At that time, balls are at rest and their KE is zero. In fact, all this KE has been transformed into elastic potential energy of balls.

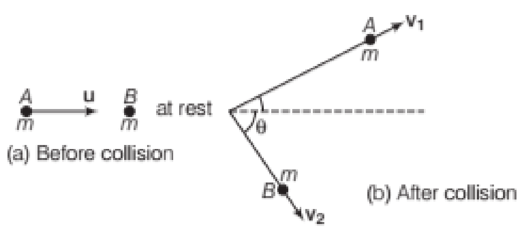
(ii) Yes, total linear momentum remains conserved during the short time of an elastic collision of two balls. The balls exert forces on one another due to which individual momenta of two balls change but total linear momentum remains conserved.

(iii) For an inelastic, collision kinetic energy is not conserved but total linear momentum is conserved even now.

(iv) As the potential energy depends only on the separation distance between the centres of balls, it means that conservative forces are in action (because PE changes due to conservative forces only). Hence, collision is surely inelastic collision.

29. Prove that when a particle suffers an oblique elastic collision with another particle of equal mass and initially at rest, the two particles would move in mutually perpendicular directions after collisions.

Sol. Let a particle A of mass m and having velocity $\mathbf{u}$ collides with particle B of equal mass but at rest. Let the collision be oblique elastic collision and after collision the balls A and B move with velocities $\mathbf{v}_1$ and $\mathbf{v}_2$ respectively inclined at an angle θ from each other.



Applying principle of conservation of linear momentum, we get

$$m\mathbf{u} = m\mathbf{v}_1 + m\mathbf{v}_2 \text{ or } \mathbf{u} = \mathbf{v}_1 + \mathbf{v}_2$$

or
$$u^2 = (\mathbf{v}_1 + \mathbf{v}_2)\cdot(\mathbf{v}_1 + \mathbf{v}_2)$$
$$= v_1^2 + v_2^2 + 2v_1v_2\cos\theta \quad \text{...(i)}$$

Again as total KE before collision = total KE after collision

$$\therefore \quad \frac{1}{2}mu^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2$$

$$\Rightarrow \quad u^2 = v_1^2 + v_2^2 \quad \text{...(ii)}$$

Comparing Eqs. (i) and (ii), we get $2v_1v_2\cos\theta = 0$

As in an oblique collision both v_1 and v_2 are finite, hence $\cos\theta = 0$

$$\Rightarrow \quad \theta = \cos^{-1}(0) = \frac{\pi}{2}$$

Thus, particles A and B are moving in mutually perpendicular directions after the collision.

30. A particle of mass m moving with an initial velocity u collides inelastically with a particle of mass M initially at rest. If the collision is completely inelastic, then find expressions for (i) final velocity of combined entity and (ii) loss in kinetic energy during collision.

Sol. (i) Let a particle of mass m moving with an initial velocity u collides inelastically with another particle of mass M initially at rest. Let after collision, the combined entity moves with a velocity v. Then, from the conservation of linear momentum, we have

$$mu + 0 = (m + M)\,v$$

$$\Rightarrow \quad v = \frac{mu}{m + M}$$

(ii) Initial kinetic energy of system before collision

$$K = \frac{1}{2}mu^2$$

and final kinetic energy of system after collision

$$K' = \frac{1}{2}(m + M)v^2 = \frac{1}{2}(m + M)\cdot\left(\frac{mu}{m + M}\right)^2$$

$\therefore$ Loss in kinetic energy during collision

$$= \frac{1}{2}\frac{m^2u^2}{(m + M)} = \frac{1}{2}\frac{m^2u^2}{(m + M)}\Delta K = K - K'$$

$$= \frac{1}{2}mu^2 - \frac{1}{2}\frac{m^2u^2}{(m + M)}$$

$$= \frac{1}{2}mu^2\left[1 - \frac{m}{M + m}\right] = \frac{1}{2}mu^2\cdot\left(\frac{M}{M + m}\right)$$

and fractional loss in kinetic energy during collision

$$\frac{\Delta K}{K} = \frac{M}{M + m}$$

ASSESS YOUR TOPICAL UNDERSTANDING

OBJECTIVE Type Questions

1. A one kilowatt motor is used to pump water from a well 10 m deep. The quantity of water pumped out per second is nearly

(a) 1kg (b) 10 kg
(c) 100 kg (d) 1000 kg

2. A molecule in a gas container hits a horizontal wall with speed 200 ms^{-1} and angle 30° with the normal and rebounds with the same speed. Which statement is true?

(a) Momentum is conserved
(b) Elastic collision
(c) Inelastic collision
(d) Both (a) and (b)

3. A particle of mass 1g moving with a velocity $\mathbf{v}_1 = 3\hat{\mathbf{i}} - 2\hat{\mathbf{j}}\ ms^{-1}$ experiences a perfectly elastic collision with another particle of mass 2 g and velocity $\mathbf{v}_2 = 4\hat{\mathbf{j}} - 6\hat{\mathbf{k}}\ ms^{-1}$. The velocity of the particle is

(a) 2.3 ms^{-1} (b) 4.6 ms^{-1}
(c) 9.2 ms^{-1} (d) 6 ms^{-1}

4. In an inelastic collision,

(a) conservation of momentum is not followed
(b) conservation of mechanical energy is not followed
(c) conservation of mechanical energy is followed
(d) None of the above

5. The power of a windmill having blade area equal to A and wind velocity equal to v is (ρ is density of air)

(a) $\frac{A\rho v^3}{2}$ (b) $\frac{A\rho v^2}{2}$ (c) $\frac{A\rho v}{2}$ (d) $A\rho v^3$

Answer

1. (b) | 2. (d) | 3. (b) | 4. (c) | 5. (a)

VERY SHORT ANSWER Type Questions

6. What is an oblique collision?

7. What happen when two identical objects moving in mutually opposite directions suffer elastic collision?

SHORT ANSWER Type Questions

8. Show that coefficient of restitution for one-dimensional elastic collision is equal to one.

9. An engine of 4.9 kw power is used to pump water from a well 50m deep. Calculate the quantity of water in kilolitres which it can pump out in one hour.

10. Is it possible for an individual to have

(i) more power and less energy
(ii) less power and more energy?

11. Prove that instantaneous power is given by the scalar product of force and velocity.

12. What are oblique collisions? Find the final velocities when two bodies of equal mass collide with each other.

LONG ANSWER Type I Questions

13. What do you meant by conservation of linear momentum?

14. Explain perfectly inelastic collision in one dimension.

15. Calculate the horsepower of a man who can chew ice at the rate of 30 g per minute.
Given 1hp = 746 W, 1 J = 4.2 J/cal and latent heat of ice = 80 cal/gm [**Ans.** 0. 225 HP]

16. What are power, instantaneous power and average power? Is there any relationship between power and impulse? Elucidate your answer?

17. A body of mass m moving with speed v collides elastically head-on with another body of mass m initially at rest. Show that moving body will come to a stop as a result of this collision.

LONG ANSWER Type II Questions

18. A mass m is moving with a speed u collides with a similar mass m at rest, elastically and obliquely prove that they will move in directions making an angle $\pi/2$ with each other.

19. A billiard ball of radius 1.5 cm moving with 1 m/s hits an identical ball at rest. If the impact parameter of the collision is 1.5 cm, find the speed and direction of each ball after the collision, assuming it to be elastic.
[**Ans.** $v_1 = (1/2)$ m/s, $v_2 = (\sqrt{3}/2)$ m/s, $\theta = 60°$]

20. A body of mass M at rest is struck by a moving body of mass m. Show that the fraction of the initial kinetic energy of moving mass m transferred to the strucked body is $4\ Mn/(m+M)^2$.

21. Define the terms elastic and inelastic collision. What is the difference between a partially, inelastic collision and a completely inelastic collision?

SUMMARY

- Work is said to be done whenever a force acts on a body and the body moves through some distance in the direction of the force. It is the dot product of force and displacement i.e. $W = \mathbf{F} \cdot \mathbf{s} = Fs\cos\theta$, where θ is the angle between **F** and **s**.
  - (i) If $\theta = 90°$; then $W = 0$,
  - (ii) $\theta < 90°$, then $W = +\text{ve}$,
  - (iii) $\theta > 90°$; then $W = -\text{ve}$
- Work done is a scalar quantity measured in newton-metre. Its dimension is $[ML^2T^{-2}]$.
- The SI unit of work is joule and the CGS unit of work is erg. 1 joule $= 10^7$ erg
- Work done by variable force F is given by $W = \int F \cdot ds =$ Area under the force-displacement curve.
  - (a) Forces are **conservative** if,
    - (i) Work done in a closed path is zero.
    - (ii) Work done is independent of path.
  - (b) Forces are **non-conservative** if,
    - (i) Work done in a closed path is not zero.
    - (ii) Work done depends upon the path.
- The energy of a body is defined as its ability of doing work. It is a scalar quantity. Like work, SI unit of energy is joule and the CGS unit is erg.
- The energy possessed by a body by virtue of its motion is called kinetic energy. The KE of a body of mass m moving with velocity v is given by KE $= \frac{1}{2}mv^2$ or $\frac{p^2}{2m}$
- **According to work-energy theorem**, the work done by the net force acting on a body is equal to the change in kinetic energy of the body. W = Change in KE $= \frac{1}{2}mv^2 - \frac{1}{2}mu^2$
- The energy possessed by a body by virtue of its position or configuration is called its **potential energy**.

  The gravitational potential energy of an object of mass m at height h from the earth's surface is given by $u = mgh$, where g is acceleration due to gravity.
- The energy associated with the state of compression or extension of an elastic (spring like) object is called elastic potential energy. If a spring is stretched through a distance x, then the elastic potential energy of the spring is given by

  $u = \frac{1}{2}kx^2$, where k is the force constant of the spring.
- **Power** is the rate of doing work. $P = \frac{\text{Energy}}{\text{Time}}$ or $\frac{\text{Work done}}{\text{Time}}$. It is a scalar quantity and it is measured in Js^{-1} or watt.
- A collision in which both the momentum and kinetic energy of the body remains conserved is called **elastic collision**.
- A collision in which only the momentum of the system is conserved but kinetic energy is not conserved is called **inelastic collision**.
- If the colliding bodies move along the same straight line path before and after the collision, then the collision is said to be one dimensional collision.
- If the two bodies do not move along the same straight line path before and after the collision, then the collision is said to be two dimensional or oblique collision.
- If two bodies of masses m_1 and m_2 moving with velocities u_1 and u_2 $(u_2 > u_1)$ in the same direction suffer head-on elastic collision such that v_1 and v_2 be their respective velocities after collision, then

  $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u_1 + \left(\frac{2m_2}{m_1 + m_2}\right)u_2$$

  and $$v_2 = \left(\frac{2m_1}{m_1 + m_2}\right)u_1 + \left(\frac{m_2 - m_1}{m_1 + m_2}\right)u_2$$
- If a body of mass m_1 moving with velocity u_1 collides with another body of mass m_2 and sticks to it (i.e. collision is perfectly inelastic collision), then the final common velocity is given by $v = \frac{m_1u_1}{m_1 + m_2}$
- In such a case loss in kinetic energy of the system is $\Delta KE = \frac{m_1m_2u_1^2}{2(m_1 + m_2)}$
- If a particle of mass m_1 moving with velocity u_1 collides with another particle of mass m_2 at rest, then after the collision the two particles move with velocities v_1 and v_2, making angles θ_1 and θ_2 with x-axis

  $m_1v_1\sin\theta_1 = m_2v_2\sin\theta_2$ (along y-axis)

  $m_1u_1 = m_1v_1\cos\theta + m_1v_2\cos\theta_2$ (along x-axis)
- The coefficient of restitution gives a measure of the degree of restitution of a collision and it is defined as the ratio of relative velocity of separation after collision to the relative velocity of approach before collision. It is denoted by e. i.e. $e = \frac{v_2 - v_1}{u_1 - u_2}$
- For a perfectly elastic collision, $e = 1$ and for a perfectly inelastic collision, $e = 0$

CHAPTER PRACTICE

OBJECTIVE Type Questions

1. The earth, moving around the sun in a circular orbit, is acted upon by a force and hence work done on the earth by the force is
(a) zero (b) + ve
(c) − ve (d) None of these

2. A force $F = -k/x^2$ $(x \neq 0)$ acts on a particle in X-direction. Find the work done by the force in displacing the particle from $x = -a$ to $x = 2a$.
(a) $3k/2a$ (b) $4k/a^2$ (c) $-3k/2a^2$ (d) $\frac{-9k}{a^2}$

3. A force of 10 N is applied on an object of mass 2 kg placed on a rough surface having coefficient of friction equal to 0.2. Work done by applied force in 4 s is
(a) 120 J (b) 240 J
(c) 250 J (d) 100 J

4. A man squatting on the ground gets straight up and stand. The force of reaction of ground on the man during the process is **[NCERT Exemplar]**
(a) constant and equal to mg in magnitude
(b) constant and greater than mg in magnitude
(c) variable but always greater than mg
(d) at first greater than mg and later becomes equal to mg

5. The potential energy, *i.e.*, $U(x)$ can be assumed zero when
(a) $x = 0$
(b) gravitational force is constant
(c) infinite distance from the gravitational source
(d) All of the above

6. What is the ratio of kinetic energy of a particle at the bottom to the kinetic energy at the top when it just loops a vertical loop of radius r?
(a) 5:1 (b) 2:3 (c) 5:2 (d) 7:2

7. Two bodies of masses m_1 and m_2 have same momentum. The ratio of their KE is
(a) $\sqrt{\frac{m_2}{m_1}}$ (b) $\sqrt{\frac{m_1}{m_2}}$ (c) $\frac{m_1}{m_2}$ (d) $\frac{m_2}{m_1}$

8. If the linear momentum is increased by 50%, then kinetic energy will be increased by
(a) 50% (b) 100% (c) 125% (d) 25%

9. How much amount of energy is liberated to convert 1 kg of coal into energy?
(a) 9×10^{16} J (b) 9×10^{15} J
(c) 3×10^{14} J (d) 4×10^{6} J

10. In a hydroelectric power station, the water is flowing at 2 ms^{-1} in the river which is 100 m wide and 5 m deep. The maximum power output from the river is
(a) 1.5 MW (b) 2 MW (c) 2.5 MW (d) 3 MW

11. In a head on elastic collision of a very heavy body moving with velocity v with a light body at rest. Then, the velocity of heavy body after collision is
(a) v (b) $2v$ (c) zero (d) $\frac{v}{2}$

12. The height attained by a ball after 3 rebounds on falling from a height of h on floor having coefficient of restitution e is
(a) e^3h (b) e^4h (c) e^5h (d) e^6h

ASSERTION-REASON

Direction (Q.Nos. 13-17) *In the following questions, two statements are given- one labelled Assertion (A) and the other labelled Reason (R). Select the correct answer to these questions from the codes (a), (b), (c) and (d) as given below*
(a) Both Assertion and Reason are true and Reason is the correct explanation of Assertion.
(b) Both Assertion and Reason are true but Reason is not the correct explanation of Assertion.
(c) Assertion is true but Reason is false.
(d) Assertion is false but Reason is true.

13. Assertion Two springs of force constants k_1 and k_2 are stretched by the same force. If $k_1 > k_2$, then work done in stretching the first (W_1) is less than work done in stretching the second (W_2).

Reason $F = k_1x_1 = k_2x_2$

14. **Assertion** If momentum of a body increases by 50%, its kinetic energy will increase by 125%.

Reason Kinetic energy is proportional to square of velocity.

15. **Assertion** Stopping distance $= \dfrac{\text{Kinetic energy}}{\text{Stopping force}}$

Reason Work done in stopping a body is equal to change in kinetic energy of the body.

16. **Assertion** Mass and energy are not conserved separately but are conserved as a single entity called 'mass-energy'.

Reason This is because one can be obtained at the cost of the other as per Einstein's equation

$$E = mc^2$$

17. **Assertion** Force applied on a block moving in one dimension is producing a constant power, then the motion should be uniformly accelerated.

Reason This constant power multiplied with time is equal to the change in kinetic energy.

Answer

CASE BASED QUESTIONS

Direction (Q. Nos. 18) *This question is case study based questions. Attempt any 4 sub-parts from each question.*

18. **PE of Spring**

There are many types of spring. Important among these are helical and spiral springs as shown in figure.

Usually, we assume that the springs are massless. Therefore, work done is stored in the spring in the form of elastic potential energy of the spring. Thus, potential energy of a spring is the energy associated with the state of compression or expansion of an elastic spring.

(i) The potential energy of a body is increases in which of the following cases?
(a) If work is done by conservative force
(b) If work is done against conservative force
(c) If work is done by non-conservative force
(d) If work is done against non- conservative force

(ii) The potential energy, i.e. $U(x)$ can be assumed zero when
(a) $x = 0$
(b) gravitational force is constant
(c) infinite distance from the gravitational source
(d) All of the above

(iii) The ratio of spring constants of two springs is 2 : 3. What is the ratio of their potential energy, if they are stretched by the same force?
(a) 2 : 3 (b) 3 : 2 (c) 4 : 9 (d) 9 : 4

(iv) The potential energy of a spring increases by 15 J when stretched by 3 cm. If it is stretched by 4 cm, the increase in potential energy is
(a) 27 J (b) 30 J
(c) 33 J (d) 36 J

(v) The potential energy of a spring when stretched through a distance x is 10 J. What is the amount of work done on the same spring to stretch it through an additional distance x?
(a) 10 J (b) 20 J (c) 30 J (d) 40 J

Answer

1. (a)	2. (a)	3. (b)	4. (d)	5. (d)
6. (a)	7. (d)	8. (c)	9. (a)	10. (b)
11. (a)	12. (d)	13. (a)	14. (a)	15. (a)
16. (a)	17. (d)			

18. (i) (b) (ii) (d) (iii) (b) (iv) (a) (v) (c)

VERY SHORT ANSWER Type Questions

19. How much work is done by mass M moving once around a horizontal circle of radius r?

20. Does the work done in moving a body depend upon how fast the body is moved?

21. Define spring constant of a spring. Give its SI unit.

22. Friction is a non-conservative force. Why?

23. What is coefficient of restitution?

24. A person walking on a horizontal road with a load on his head does no work. Why?

25. Find the work done in moving a particle along a vector $\mathbf{s} = (2\hat{\mathbf{i}} - 3\hat{\mathbf{j}} + \hat{\mathbf{k}})$ metre, if applied force is $\mathbf{F} = (\hat{\mathbf{i}} - 2\hat{\mathbf{j}} + 3\hat{\mathbf{k}})$N. [Ans. – 6 J]

SHORT ANSWER Type Questions

26. Calculate the kinetic energy of a body of mass 0.1 kg, if its linear momentum is 20 kg-m/s. [**Ans.** 2000 J]

27. Momentum of a body is doubled. What is the percentage increase in kinetic energy ?[**Ans.** 100%]

28. Find the average frictional force needed to stop a car weighing 800 kg running with an initial speed of 54 km/h with a distance of 25 m. [**Ans.** 3600 N]

29. A steel spring of spring constant 150 N/m is compressed from its natural position through a mud wall 1m thick, the speed of bullet drops to 100 m/s. Calculate the average resistance of the wall. Neglect friction of air. [**Ans.** 0.12 J]

30. Two balls of mass *m*, each moving in opposite direction with a speed *v* collide head-on with each other. How will they move after collision, assuming it to be perfectly elastic?

LONG ANSWER Type I Questions

31. About 4×10^{10} kg of matter per second is converted into energy in the central core of the Sun. What is the power output of the Sun? [**Ans.** 3.6×10^{27} W]

32. A fast moving neutron make a head-on elastic collision with a stationary deuteron. What fraction of its initial kinetic energy is lost by the neutron during collision? $\left[\textbf{Ans. } \frac{8}{9}\right]$

33. Briefly describe the mechanism of the functioning of heavy water as moderator in nuclear reactors.

34. 1 mg of uranium is completely destroyed in an atomic bomb. How much energy is liberated? [**Ans.** 9×10^{10}]

35. A block of mass 1.2 kg moving at a speed of 20 cm/s collides head-on with a similar block kept at rest. The coefficient of restitution is 3/5, find the loss of kinetic energy during collision. [**Ans.** 7.7×10^{-3} J]

36. (i) What is the power of a centripetal force in circular motion?

(ii) A pump can take out 7200 kg of water per hour from a well 100m deep. Calculate the power of the pump, assuming that its efficiency is 50%. ($g = 10\ ms^{-2}$) [**Ans.** 4 kW]

(iii) An elevator which can carry a maximum load of 1800 kg is moving up with a constant speed of 2m/s. The frictional force opposing the motion is 4000 N. Determine the maximum power delivered by the motor to the elevator in horse power. [**Ans.** 59 hp]

37. (i) Define the term instantaneous power and write its relation with velocity.

(ii) A machine gun fire 240 bullets per minute. If the mass of each bullet is 10 g and the velocity of the bullets is 600 ms^{-1}, then find power (in kW) of th gun. [**Ans.** 7.2 kW]

(iii) A motor pumps up 1000 kg of water through length of 10 m in 5s. If the efficiency of the motor is 60%, then calculate the power of the motor in kilowatt.

LONG ANSWER Type II Questions

38. Two balls *A* and *B* of masses 0.3 kg and 0.2 kg respectively are moving along positive *x*-axis and negative *x*-axis with velocity 20 m/s. They collide and thereafter move in the direction opposite to their original direction. Find the velocity of *A* and *B* after collision. Also, calculate total KE of the balls before and after collision. [**Ans.** 1.0 J]

39. A projectile proton with a speed of 500 m/s collide elastically with a target proton initially at rest. The two protons then move along perpendicular paths, with the projectile path at 60° from the original direction. After collision, what are the speed of (i) the target proton? (ii) The projectile proton? [**Ans.** 433 m/s, 250 m/s]

40. A body of mass 2 kg makes an elastic collision with another body at rest and continuous to move in the original direction but one-fourth of its original speed. (i) what is the mass of the other body ? (ii) what is the speed of the two body centre of mass if initial speed of 2 kg body was 4.0 m/s? [**Ans.** 1.2 kg, 2.5 m/s]